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ON THE PERIOD OF THE SOLAR SPOTS.

By Simon Newcomb.

In discussing periodic phenomena in which the times of recurrence of a given phase are subject to irregularities two hypotheses may be made. One is that underlying the periodic phenomena which we observe, there is a primary cause going through a perfectly uniform period; but that, on the action of this cause are superseded irregular actions which may delay or accelerate the occurrence of a phase without affecting the primary cause. When this is the case we shall have a series of perfectly equidistant normal epochs for the recurrences of the same phase, and the observed deviations from these epochs will be in the nature of separate and independent accidental errors. That is to say, if P be the true value of the normal period then, at the end of n periods, however great n may be, the time of occurrence of the phase will differ from $n\,P$ only by a small quantity $\pm\epsilon$ indicating the irregularity in the general mean. This value of ϵ will be the same, no matter how great n may be.

As an example of this we may take the hypothesis that the variations in the solar spots are due to the action of some planet, say *Jupiter*, which acts differently in different parts of its orbit,

but always produces the same action when it returns to the same point. Then the normal phase of the solar spots would always be that corresponding to the longitude of the planet, and the deviations from this phase would be in the nature of accidental irregularities.

The other hypothesis is that, while there is still a certain normal mean period, this period is nevertheless subject to change in such a way that, if a phase is once accelerated, the advance thus produced will go on indefinitely into all subsequent phases. For example, if the maximum of spot activity indicates a state of things leading in a regular way to a following minimum, then, if this maximum is accelerated, the occurrence of the minimum would be equally accelerated. These accidental accelerations or retardations being supposed purely accidental would follow the law of accidental error; that is to say, if $\pm\epsilon$ be the probable acceleration or retardation in a single period, then, at the end of n periods the probable deviation from the normal would be $\pm\epsilon \times \sqrt{n}$. As n increased there would be no limit to the possible deviation of the phase.

In deciding which of these two hypotheses applies to a given case we are met with the difficulty that we cannot determine the normal period except by the observations themselves. Consequently, although at the end of n periods the phase should be accelerated or retarded by any amount, we should be obliged, in determining the mean period, to divide the excess or deficiency among all the periods so that, apparently, we should fail to get evidence of the accumulation. But probable evidence would still be obtainable if the number of consecutive periods observed was great. We might then divide the whole period of observation into a certain number, say two or three, of equal parts. If n be the number of periods in each of these parts, then each part would probably differ from the normal length by the mean amount $\pm\epsilon\sqrt{n}$. These variations being quite independent of each other, the probable differences between the accumulated errors of any two parts would be $\pm\epsilon\sqrt{2n}$.

If, in any special case, a difference between the accumulated

lengths was found exceeding the probable deviations, we should have strong evidence that the period was of the second class. In the contrary case the evidence would be more or less probable according to the number of periods, but never quite conclusive.

Maxima and minima of solar spots have been derived by Wolf with more or less certainty through about twenty-five full periods. It seems possible to decide whether or not there is a normal period underlying the changes of activity which they exhibit.

The method of determining a period of a varying phenomenon from the observed phases is worthy of consideration. A plausible method frequently applied is this: some definable phase is seen to occur at the n epochs $t_1, t_2, \ldots t_n$. Then, the observed periods will be $t_2 - t_1$; $t_3 - t_2$; .... $t_n - t_{n-1}$. It is common to take the mean of these periods as the normal period to be derived from the observations. But we readily see that the sum of all these periods is nothing but $t_n - t_1$; that is to say, the mean which we get is nothing more than the extreme time divided by the number of periods. Thus we completely ignore the periods which might be derived from all the intermediate observations. It is clear enough that these intermediate periods are worthy of consideration. If, for example, there were twenty recurrences, the result following from the interval between the second and the nineteenth would fall little short in accuracy from the comparison of the first and twentieth. In fact, the several values of the period,

$$\frac{t_{20} - t_1}{19}\,;\quad \frac{t_{19} - t_2}{17}\quad \ldots\ldots,$$

would be so many independent determinations of continually diminishing weight, of which all but the first are ignored in the method in question.

If the determinations of the epochs were all of equal weight the best result would be obtained by the weighted mean of the several determinations thus found. But the general method is to proceed by a least square solution which shall give an

equidistant set of times differing as little as possible from the observed times.

In applying this method to the determination of the period of the solar spots I shall depend mainly upon the epochs derived by Rudolf Wolf so far as they go. The results of his investigations up to 1872 are found in his *Handbuch der Mathematik*, etc., Zurich, 1872, Vol. II, page 296, etc. His researches in detail are found scattered through his *Astronomische Mittheilungen*. For more recent phases and for the correction of one or two of Dr. Wolf's last phases, I have depended upon the Greenwich observations. Since 1874 photographs of the Sun have been taken regularly at Greenwich and at some points in India and Mauritius. These are carefully measured on a uniform plan, and the results published in the annual volumes of Greenwich observations. In this way the mean daily spot-area of the Sun is given from month to month and year to year, with a multitude of details useful in investigating the question of their behavior. Thus we have in all a series of twenty-six observed maxima, extending from 1615 to 1893, and an equal number of minima, from 1610 to 1889.

I have not, however, depended entirely on these. In cases where a continuous series of observations are available, maxima and minima are not the phases which can be determined with the greatest precision. I have therefore included what we may call half-tide phases, increasing and decreasing. The epochs of these phases I have determined in the following way: We have at hand, from year to year, annual numbers expressive of the mean spot activity in each year. In the case of the Greenwich observations this is simply the mean daily spot area. In the case of the older observations it is something a little different, depending largely on the number of spots, but it is not to be expected that there will be any great differences between the two measures of activity. Half the sum of the numbers corresponding to the year of minimum and the following maximum, or *vice versa*, gives the number corresponding to a certain mid-phase. From the annual numbers, assumed to correspond to the middle of each

year, is found a time corresponding to this mid-phase. Such a time would, from the nature of the variation, be subject to less uncertainty than the time of a maximum or minimum. In forming these mid-phases I use the numbers of Dr. Wolf so far as they are found in his *Mittheilungen* and in the spot areas derived at Greenwich.

In the case of any one phase our method of proceeding is now this: We assume a certain epoch, as near as possible to one of the phases, and a certain provisional period. With this provisional epoch and period we form an arithmetical series expressing the provisional times at which the phase would occur were the assumed epoch and period correct. Each observed epoch gives rise to an equation of condition of the form

$$x + by = \Delta t$$

in which x is the correction to the assumed zero phase, y the correction to the length of the period, and Δt the discrepancy between the observed and computed epochs. The least square solution of the equations thus formed gives the concluded results from each series of epochs.

The question of the relative weights to be assigned to the observed phases is a difficult one. In the observed time of a phase two classes of errors enter. One is that of the observations themselves, the other the irregularities of the actual phase. In the case of the Greenwich results the former error is almost evanescent. The magnitude of the latter or real irregularity is shown by the fact that the epoch of maximum or minimum may be a year different for the two solar hemispheres. As an example of this, extreme cases occurred in the years preceding and following the maximum of 1883 and the minimum of 1889 The actual mean irregularity in the best determined phase would seem to be at least $0^y.4$, perhaps $0^y.5$. On this actual irregularity is superposed the uncertainty rising from the irregularity of the observations. This last is such that Dr. Wolf sometimes assigns a probable error as great as two years to the times of a phase derived from the observations. In assigning weights,

however, I do not deem it advisable to use so great a difference as would be indicated by these probable errors.

A preliminary solution was first made of each of the four phases, from which it would be inferred that the normal period was quite near to 11.13 years. This period, however, and the epochs derived in connection with it were considered only as provisional values to be corrected by a definitive solution. The deviations are, however, of such a nature that they may be regarded as accidental errors. In order to save space I omit this preliminary solution as well as the numbers not necessary to judge the results. The actual numbers on which my conclusions are based are presented as follows, in order that the method of derivation may be most easily seen.

In the first column is given an equidistant series of computed epochs for each of the four phases, maximum, minimum, mid-phase rising and mid-phase falling.

In the column following is given the observed epoch of the phase.

Next follows the weight assigned to this determination. In the case of the older observations this weight was generally the same as that of the preliminary solution. The comparison of the latter with observation showed, however, that the modern observations were entitled to a relative weight nearly double that assigned to them in the first solution. Their weights were, therefore, increased.

The next column gives the observed corrections to the computed phase, which is, in fact, merely the difference of the first two columns. Next we have in column *b* the number of the period from an epoch quite near the weighted mean of all the epochs, divided by ten.

A few remarkable deviations demand attention. It would seem that during the decade 1670–1680 there was a considerable retardation in the phases. This might be taken to indicate that the second hypothesis was the correct one, and that the actual period was subject to acceleration and retardation. But we find that, during the two following decades, this seeming retardation

Maxima					Minima				
Comp.	Obs.	Wt.	*n*	*b*	Comp.	Obs.	Wt.	*n*	*b*
1615.36	15.5	3	+0.1	−1.8	1610.79	10.8	5	0.0	−1.8
26.49	26.0	5	−0.5	−1.7	21.92	19.0	2	−2.9	−1.7
37.62	39.5	5	+1.9	−1.6	33.05	34.0	3	+1.0	−1.6
48.75	49.0	3	+0.2	−1.5	44.18	45.0	3	+0.8	−1.5
59.88	60.0	1	+0.1	−1.4	55.31	55.0	1	−0.3	−1.4
71.01	75.0	1	+4.0	−1.3	66.44	66.0	1	−0.4	−1.3
82.14	85.0	3	+2.9	−1.2	77.57	79.5	1	+1.9	−1.2
93.27	93.0	1	−0.3	−1.1	88.70	89.5	1	+0.8	−1.1
1704.40	05.5	5	+1.1	−1.0	99.83	98.0	1	−1.8	−1.0
15.53	18.2	5	+2.7	−0.9	1710.96	12.0	3	+1.0	−0.9
26.66	27.5	5	+0.8	−0.8	22.09	23.5	3	+1.4	−0.8
37.79	38.7	5	+0.9	−0.7	33.22	34.0	3	+0.8	−0.7
48.92	50.0	5	+1.1	−0.6	44.35	45.0	3	+0.6	−0.6
60.05	61.5	10	+1.4	−0.5	55.48	55.5	10	0.0	−0.5
71.18	69.9	15	−1.3	−0.4	66.61	66.5	5	−0.1	−0.4
82.31	79.5	10	−2.8	−0.3	77.74	75.8	5	−1.9	−0.3
93.44	89.0	10	−4.4	−0.2	88.87	84.8	5	−4.1	−0.2
1804.57	04.0	5	−0.6	−0.1	1800.00	98.5	5	−1.5	−0.1
15.70	16.8	10	+1.1	0.0	11.13	10.5	5	−0.6	0.0
26.83	28.7	10	+1.9	+0.1	22.26	23.2	5	+0.9	.1
37.96	37.2	20	−0.8	0.2	33.39	33.8	15	+0.4	.2
49.09	48.6	20	−0.5	0.3	44.52	44.0	20	−0.5	.3
60.22	60.2	20	0.0	0.4	55.65	56.2	20	+0.6	.4
71.35	70.9	20	−0.4	0.5	66.78	67.2	20	+0.4	.5
82.48	83.7	20	+1.2	0.6	77.91	78.8	20	+0.9	.6
93.61	93.6	20	0.0	0.7	89.04	89.4	20	+0.4	.7

Mid-phase rising					Mid-phase falling				
Comp.	Obs.	Wt.	*n*	*b*	Comp.	Obs.	Wt.	*n*	*b*
1757.79	58.7	1	+0.9	−.8	1751.69	52.0	2	+0.3	− .9
68.92	68.3	2	−0.6	−.7	62.82	62.8	2	0.0	− .8
80.05	77.5	1	−2.5	−.6	73.95	72.2	1	−1.8	− .7
91.18	86.2	1	−5.0	−.5	85.08	81.8	2	−3.3	− .6
1802.31	....	..	.....	....	96.21	....	..	.....	.. ..
13.44	15.0	5	+1.6	−.3	1807.34	....	..	.....	.. ..
24.57	26.5	5	+1.9	−.2	18.47	19.5	3	+1.0	− .3
35.70	35.8	10	+0.1	−.1	29.60	31.8	6	+2.2	− .2
46.83	46.7	20	−0.1	0	40.73	40.0	10	−0.7	− .1
57 96	58.4	20	+0.4	+.1	51.86	52.5	10	+0.6	.0
69.09	69.3	20	+0.2	.2	62.99	62.9	15	−0.1	+ .1
80.22	80.7	20	+0.5	.3	74.12	73.2	15	−0.9	.2
91.35	91.8	20	+0.4	.4	85.25	86.1	15	+0.8	.3
					96.37	96.0	15	−0.4	.4

was lost. More remarkable yet is the acceleration about 1790, which, it will be seen, seems to affect all the phases. But, again, during the following two or three decades, this acceleration is changed into a retardation. I was at first disposed to think that these perturbations of the period might be real, but, on more mature consideration, I think they are to be regarded as errors rising from the imperfection of the record. The derivation of any exact epoch requires a fairly continuous series of observations made on a uniform plan. If we compare and combine the results of observations made in any irregular or sporadic way it may well be that the actual changes are masked by the apparent changes due only to these imperfections.

A curious case of this is afforded by a comparison of Carrington's results with the contemporaneous observations of Schwabe. During the years 1856–1859, which immediately followed a minimum, the number of new groups noted by each observer was nearly the same as we should expect, but, as the maximum approached, Carrington had decidedly more groups than Schwabe.

On this consideration was based the great disparity of weights which were finally assigned to the several observed phases.

A curious anomaly is the preponderance of positive corrections given by nearly all the maxima from the beginning to 1760. For this reason the period as it comes out from the maxima is smaller than that from the other phases. The fact appears to be that, while modern observations show that the maximum follows the minimum by less than five years, and between six and seven years are required to again fall to the minimum, the older observations seem to place the two phases nearly equidistant. I regard this only as resulting from the accidental errors of the observations, as we can scarcely suppose a change in the law of variation to have occurred.

In the first of the definitive solutions I include the remarkably discordant epochs about 1780–1790, on the ground that previous observations might well have been affected with the same kind of an error arising from the imperfect continuity of the record. With the weights as assigned we have the following

normal equations and solutions for y. Here ϵ_1 is the mean error for weight 1 as derived from the residual values of n, and ϵ is the mean error of the value of y.

MAXIMA.

$$\begin{array}{l} 227x - 15y = -7^y.1 \\ -15 + 106 = -30.7 \end{array} \quad \epsilon_1 = \pm 4^y.6.$$

Solution: $y = -0^y.297 \pm 0^y.43$.

MINIMA.

$$\begin{array}{l} 185x + 7y = +16^y.7 \\ 7 + 83 = +23.7 \end{array} \quad \epsilon_1 = \pm 2^y.8.$$

Solution: $y = +0^y.220 \pm 0^y.31$.

MID-PHASE RISING.

$$\begin{array}{l} 125x + 13y = +40^y.0 \\ 13 + 9 = +7.4 \end{array} \quad \epsilon_1 = \pm 2^y.5.$$

Solution: $y = +0^y.42 \pm 0^y.91$.

MID-PHASE FALLING.

$$\begin{array}{l} 96.0x + 6.6^y = -1^y.8 \\ 6.6 + 9.3 = +0.3 \end{array} \quad \epsilon_1 = \pm 2^y.7.$$

Solution: $y = +0^y.05 \pm 0^y.92$.

Applying $y \div 10$ as a correction to the provisional value $11^y.13$ of P we have the following four results and combination:

Maxima - - - - -	$P = 11^y.100 \pm 0^y.043$	
Minima - - - - - -	$11.152 \pm .031$	
Mid-phase R. - - - -	$11.172 \pm .091$	
Mid-phase F. - - - -	$11.135 \pm .092$	
Mean period - - - -	$11.136 \pm .023$	(A)

The striking abnormality of the phases between 1781 and 1792 is shown by the fact that they contribute about one third of the whole sum of the squares of the errors. We are therefore justified in at least undertaking a solution in which these discordant epochs, one for each phase, are dropped out. When this is done the equations of condition and solution will be:

$$\text{Maxima} \quad \begin{array}{l} 217x - 13.3y = +37.0 \\ 13 + 105.3 = -39.5 \end{array} \quad \epsilon_1 = \pm 3.8.$$

Result: $y = -0.354 \pm 0.37$.

$$\text{Minima}\quad \begin{array}{l} 180.0x + 7.8y = +29.2 \\ 7.8 + 83.0 = +19.8 \end{array} \quad \epsilon_1 = \pm 2.2.$$

Result: $y = +0.223 \pm 0.24$.

$$\text{Mid-phase R.}\quad \begin{array}{l} 124.0x + 13.7y = +45.0 \\ 13.7 + 8.8 = +4.9 \end{array} \quad \epsilon_1 = \pm 2.1.$$

Result: $y = -0.013 \pm 0.80$.

$$\text{Mid-phase F.}\quad \begin{array}{l} 94.0x + 7.8y = +4.8 \\ 7.8 + 8.6 = -3.7 \end{array} \quad \epsilon_1 = \pm 2.5.$$

Result: $y = -0.512 \pm 0.89$.

We therefore have

From Maxima - - - -	$P = 11^y.095 \pm 0^y.037$		
From Minima - - - - -	$11\ .152 \pm$	$.024$	
From Mid-phase R. - - -	$11\ .129 \pm$	$.080$	
From Mid-phase F. - - -	$11\ .079 \pm$	$.089$	
Mean - - - - - -	$11\ .132 \pm$	$.018$	(B)

The difference between the two results A and B is much less than the probable error of either. It is not therefore necessary, in order to fix the period, that we should decide between them. Omitting useless decimals we conclude that the normal period of the solar spots is

	$P = 11^y.13 \pm 0^y.02$
Rise from minimum to maximum	$4\ .62$
Fall from maximum to minimum	$6\ .51$

We may now consider the important question whether there is any evidence of an accumulation of accidental irregularities in the course of the successive cycles. One way of doing this would be to find the mean period separately for the first half of the series, and the second half. But then the result would depend almost entirely on the retention or omission of the abnormal residuals. I therefore divide the whole period of observation into three parts, the first extending from 1610 to 1720, the second from 1720 to 1820, and the third from 1820 to the present time. Taking the weighted mean residuals for each of these periods, and including all the epochs, we find the mean deviations to be

	From Maxima	From Minima
1st period - - - - -	$+1^y.2$	$+0^y.1$
2d period - - - - - -	$-0\ .9$	$-0\ .8$
3d period - - - - -	$+0\ .1$	$+0\ .2$

It would thus appear that near the mid-epoch there was an apparent systematic acceleration amounting to somewhat more than a year. But this conclusion rests on the hypothesis that there is nothing abnormal in the great residuals. If we exclude those between 1775 and 1790, in which the evidence of abnormality is so strong, the deviations of the middle period will be:

Maxima $+ 0^y.3$; Minima $- 0^y.1$.

The deviation from uniformity is now reduced to $0^y.3$, a quantity markedly less than the probable variation of a single period. If each period were subject independently to an accidental error liable to accumulate, the deviation at the mid-epoch would be between 1.5 and 2 years. Were this the case a more systematic character would be seen in the residuals for the middle period. The contrast between the sudden deviations in the residuals of the doubtful period and the small ones of the recent well observed epochs make it almost certain that the errors between 1770 and 1800 are due to imperfections of the record. I therefore consider the most probable conclusion to be that there is no accumulation of accidental errors in the course of successive cycles, and that the first of the two hypotheses set forth in the beginning is the correct one. If during each short period, say one year, the progress of a cycle was measured by the apparent spot activity of the time, there would be an accumulation of the kind we have been looking for. Our final conclusion is therefore this:

Underlying the periodic variations of spot activity there is a uniform cycle, unchanging from time to time and determining the general mean of the activity.

Whether the cause of this cycle is to be sought in something external to the Sun, or within it; whether, in fact, it is in the nature of a cycle of variations within the Sun, we have, at present, no way of deciding.

It would seem from what precedes that a revision of the conclusions to be drawn from the observations of Sun-spots during the interval of 1775–1790 is very desirable.

The preceding conclusions rest upon a discussion of four

separate phases, expressing the entire general degree of spot activity. The question may arise whether the singular change which takes place in the distribution of the spots in latitude at the time of minimum may not give a more definite series of epochs than those of the phases we have considered. The best defined of these additional epochs seems to be that at which, about the time of a minimum, spots begin to show themselves in very high latitudes. I find that we can regard this phenomenon as a fairly well-defined one, the mean epoch of, we may say, the first four spots being taken. A comparison of these phases shows, however, that they are even less equidistant than the epochs of minimum. I find the following dates to be thus defined:

First spot	Mean of first four
1856.4	1856.6
1867.2	1867.3
1879.3	1879.5
1889.0	1889.4

The conclusion seems to be that here the phenomenal phase which we observe is not definitely and invariably connected with the exact cycle of change, the existence of which we have shown to be so probable. It may, indeed, well happen that the minimum which is derived from a series of phenomena extending over two or three years will be better connected with the cycle than with a single phenomenon like that in question.

I have remarked that the epochs of a phase derived independently from the spot activity in the northern and southern solar hemispheres separately may differ by a year from each other. It was remarked by Spörer that the phases in the southern hemisphere preceded those in the northern. A mere glance at the Greenwich numbers, however, will show that this is not a general rule for at least the maximum and minimum. I have endeavored to settle this question by deriving the epochs of mid-phase raising and falling, in a rough way, from the two hemispheres separately. The results are as follows:

	North	South	N—S
Rising phase - - - -	1859.2	1858.1	+1.1
Falling " - -	62.5	62.7	—0.2
Rising " - - - -	69.4	69.2	+0.2
Falling " - -	73.2	73.5	—0.3
Rising " - - - -	80.5	82.1	—1.6
Falling " - -	85.7	85.9	—0.2
Rising " - - - -	91.2	91.3	—0.1
Mean - - - - - - - - -			—0.16

It will be seen that this very small mean difference arises from the large difference in the rising phase 1880–1882. There seems to have been an abnormal delay in the increase of the spottedness of the southern hemisphere during the years 1880–1881. Our general conclusion is, therefore, that there is no systematic difference between the phases on the two hemispheres, but that those of each hemisphere are separately subject to considerable irregularities.

Spörer has also pointed out that the spottedness of the southern hemisphere was, from observations of Carrington and himself, greater than that of the northern. This is shown to be the case in a yet greater degree by the more recent Greenwich observations. We now have four cycles through which a comparison may be made. For the two cycles from 1856 to 1877 I have used Spörer's numbers expressing the frequency of spots; from 1878 onward I use the Greenwich mean daily areas. The sum total for the two hemispheres is as follows:

	North	South	S—N	Authority
1856 to 66 - - -	3442	3680	238	Spörer
67 to 77 - -	3247	3698	451	Spörer
78 to 88 - - -	2479	3434	955	Greenwich
89 to 98 - -	3198	3908	710	Greenwich

The difference is several times larger than the probable accumulation of accidental irregularities and shows with fair conclusiveness that, for at least four cycles, the spottedness of the southern hemisphere has been one fifth greater than that of the northern. Whether this is a permanent feature of solar activity is an interesting question which only the future can decide.

For convenient reference we give the epochs of some maxima and minima derived from the concluded theory:

Maxima	Minima
1871.52	1878.03
1882.65	1889.16
1893.78	1900.29
1904.91	1911.42
1916.04	1922.55
1927.17	1933.68

ON AN APPARATUS FOR THE LABORATORY DEMONSTRATION OF THE DOPPLER-FIZEAU PRINCIPLE.

By A. BÉLOPOLSKY.

An apparatus for this purpose was suggested by me in the year 1894,[1] and since then I have made numerous attempts to construct it. Thanks to a grant of three hundred dollars which I received early in 1898 from the "Elizabeth Thompson Science Fund," I have succeeded in my attempts this year. The other necessities, such as the spectroscopic apparatus, the electric current, etc., were supplied me by the Pulkowa Observatory. I express here my thanks to both of these scientific institutions.

The principle of the apparatus is as follows: If a source of light is reflected in two nearly parallel mirrors, the distance S of the nth reflection from the source itself may be expressed as

$$S = h + 2nx + l,$$

where h is the distance from the source to a plane midway between the two mirrors, l is the distance of the image from the same plane after n reflections, and x is the distance between the two mirrors. If we differentiate this expression according to t we obtain

$$\frac{dS}{dt} = 2n\frac{dx}{dt}.$$

$\frac{dx}{dt}$ is the velocity of the mirror. We see that although $\frac{dx}{dt}$ may itself attain no very large value, $\frac{dS}{dt}$ will be $2n$ times larger. If the mirror had, for instance, a velocity of 50 meters per second, its image after ten reflections would move with a velocity of $2 \times 10 \times 50 = 1000$ meters per second.

We can also show that the wave-length of a homogeneous beam of light after n reflections from plane, moving mirrors

[1] *Mem. Spettr. Ital.*, **23**; *A. N.*, No. 3267.

alters according to the direction of the motion.[1] We obtain the following expression for the wave-length λ after one reflection from a moving mirror :

$$\lambda_1 = \lambda_0 \left(1 \pm \frac{2v}{V} \cos \psi\right),$$

where λ_0 is the normal wave-length, v the velocity of the mirror, V the velocity of light, ψ the angle between the direction of motion of the mirror and the normal to its surface. If the beam is successively reflected from several mirrors, we shall obtain the following wave-lengths, provided that all the mirrors have the same velocity and that ψ is constant:

$$\text{After the 1st reflection, } \lambda_1 = \lambda_0 \left(1 \pm \frac{2v}{V} \cos \psi\right).$$

$$\text{After the 2d reflection, } \lambda_2 = \lambda_1 \left(1 \pm \frac{2v}{V} \cos \psi\right).$$

$$\text{After the } n\text{th reflection, } \lambda_n = \lambda_{n-1} \left(1 \pm \frac{2v}{V} \cos \psi\right).$$

Hence we obtain, with sufficient approximation,

$$\lambda_n = \lambda_0 \left(1 \pm \frac{2nv}{V} \cos \psi\right).$$

The sign depends upon the direction of v. With a large enough value of n, $\lambda_n - \lambda_0$ will have an appreciable value, even if v remains comparatively small.

The apparatus must therefore move at least two mirrors as rapidly as possible in opposite directions. In its simplest construction it would consist of two wheels, like those of a water wheel, each rotating rapidly and carrying several mirrors. The axes of the wheels are so connected by gears that each pair of mirrors will come into a position near to parallelism at the same time. The wheels are of aluminum, of 250 mm diameter, and each carries eight silvered mirrors of size 20 × 105 × 3 mm. The mirrors are so regulated by five adjusting screws that a beam falls upon the slit of a powerful spectrograph after n reflections by all of the eight pairs. The support of the wheels is of cast iron and weighs 175 pounds. Each wheel is placed on the

[1] KETTELER, *Astronomische Undulationstheorie.*

common shaft between two electric motors, of which there are four in all. With 50 volts and from 1.5 to 2 amperes per motor, they should make 6000 revolutions per minute. Two rheostats are used to introduce gradually the current from the storage batteries, and two switches, one for each pair of motors, permit a change in the direction of rotation as desired.

The shafts of the motors are nearly parallel, but displaced somewhat sidewise in order to make room for the beam before incidence and after repeated reflections. The apparatus is mounted upon a very solid wooden table A rather poor heliostat reflects the sunlight upon a slit 10 × 20 mm in front of the apparatus.

It soon appeared that the brightness of a beam undergoing repeated reflections falls off very much, and a spectrograph of large light-power is necessary for the production of spectrograms when moving mirrors are used. For this I employed three compound prisms, set at minimum deviation for λ 433, a collimator of 1.5 meters focus, and a camera of 1.75 meters, both in wooden mountings, the prism-box being of steel, however. The whole spectrograph rests upon four long wooden screws with lock-nuts. The stability of the apparatus is increased by weights, in all amounting to some 280 pounds, which are placed at different points upon the spectrograph.

The slit is at a distance of about a meter from the mirror which reflects the light last; and either half of the slit may be employed in turn by means of a device placed directly in front of the slit. A cylindrical condenser is introduced between this device and the apparatus. The slit is observed by the light reflected from the first surface of the first prism. A diaphragm is placed closely over the collimator objective in order to see that the rays pass through the objective centrally.

Directly in front of the plate are two slides, movable from outside, which permit the exposure of any desired portion of the plate, so that the central part of the plate can be covered up and the edges exposed to the light, or the reverse. The plate-holder can be pressed into position by a screw, and has a strong spring.

With this arrangement the experiment may be made as follows:

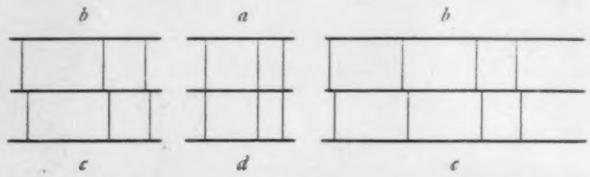
I. (*a*) One half of the slit open, say the upper half; mirror at rest; central part of plate exposed.

(*b*) Central part of plate covered, edges uncovered; mirror moving in one direction. Expose.

II. (*c*) The other half of the slit is now opened, while the first is covered; central part of plate covered, edges free; exposure made with mirrors rotating in the opposite direction.

(*d*) Mirrors at rest; central part of plate free; edges covered. Expose.

In this way we obtain upon a single plate the following spectra: Two spectra at the center of the plate, one above the



other, from mirrors at rest, to check the stability of all parts of the apparatus during the experiment; four spectra, two on each side of the central part, which should exhibit the displacements of the lines, two giving a displacement toward red, and two that toward violet.

The figure gives a schematic representation of the spectra obtained, the letters indicating the order of the spectra. *a* and *d* are the spectra for control of the stability; *b* and *c* the displaced spectra; hence the plate shows a double displacement of the lines.

The region of spectrum from $\lambda 438$ to $450 \mu\mu$ was employed, as the violet region was very faint in the reflected light (since the silvering of the mirrors transmits the violet rays). The lines found on the plate serve to compute the coefficient K for converting the measured displacements into kilometers per second.

To show that the dispersion alters very little on different plates, I give the measured distance in revolutions of the micrometer between the two lines λ4456.24 and λ4425.63, viz., January 27, 33.849 rev.; July 5, 33.877 rev.; August 9, 33.867 rev.

For these plates the following values of the coefficient K for λ4444.18 were computed by the method of least squares:

1900	June 27,	$\log K =$	1.7878
	July 6,		1.7826
	Aug. 9,		1.7871

If the magnitude of the displacement of the lines was 0.010 rev., a difference of one unit in the second place of log K would make a difference of only 14 meters in the calculated velocity. It would therefore be permissible to use the mean value of log K $=1.7858$ for all my plates.

The stellar spectrograph of this Observatory gives a precision seven times smaller in the determination of velocities in the line of sight; and since it has been shown that the velocity for stars of the second type can be determined with a probable error of ± 2 km per second for each plate, we might expect that this spectrograph could give a probable error of a few hundred meters.

The preliminary experiments, which have been in progress since the completion of the apparatus in April, have shown that a very long exposure is required to obtain a measurable spectrogram when the light is repeatedly reflected by moving mirrors. Thus sunlight reflected eight times required more than an hour, and more than two hours were necessary for two spectra. So long an exposure as this is hardly possible, for the summer sky is seldom free from clouds for so long a time, and variations in temperature and other causes may produce large changes in the parts of the spectrograph. I convinced myself that under favorable conditions no displacements of the spectral lines occurred, as was shown by two adjacent spectrograms taken two and one half hours apart.

To indicate the large loss by reflections from moving mirrors,

I may say that with the mirrors at rest, quite strong spectrograms could be obtained with 2 seconds' exposure.

Thereafter I accordingly employed only the sixth reflection, which cut the exposure time down to 30 minutes for each spectrum.

I made use of a speed indicator for determining the velocity of rotation of the apparatus; and also employed the acoustical method of estimating the pitch when a piece of paper was held against the gears. These determinations yielded the following average results:

With a current of 4½ amperes (rheostat cut out) there were 2016 revolutions in 63 seconds, or 32 per second. With the same circuit, at another time, the pitch was estimated as *La* of the third octave, corresponding to 1740 vibrations per second. The wheel had 49 teeth, so that there were 35 revolutions per second. With 7¼ amperes there were 1512 revolutions in 34 seconds, or 44 per second. At another time 505 revolutions were counted in every 11½ seconds, giving the same number of revolutions per second as above. During the whole time of the experiment the ammeter showed no variations of over ¼ ampere, whence we may infer that the rotation of the dynamos was very constant.

The breadth of the mirrors being 20 mm, the largest diameter between the edges of two mirrors standing 180° from each other was 230 mm, and the least was 190 mm, whence it follows that with the sixth reflection and 32 revolutions per second the limits of the linear velocity were 276 and 230 meters per second. With 44 revolutions per second they were 389 and 318 meters per second. It is perhaps for this reason that the lines upon the spectrogams taken with rotating mirrors have a broader appearance than those taken with mirrors at rest.

The spectrograms were measured with a micrometer screw 65 mm long attached provisionally to the microscope, as the regular screw of 35 mm was too short for the purpose; 199.4 divisions of the head corresponded to one millimeter.

The plates were always so placed under the microscope that

the readings of the head increased from red to violet. Only that half of the plate toward the violet was measured, as the length of the spectrograms was 100 mm. Settings (usually four or five on each line) were first made on the lines of the upper spectrum, then on those of the lower. The direction of the rotation of the mirrors, whether receding (−) or approaching (+) each other, is marked on the plate. I have obtained measurable plates only since June 27. The small number secured since that time is explained in part by the unfavorable weather and in part by the fact that I was also making spectrograms of the Sun with the 30-inch refractor. The measures of the plates and explanatory remarks follow.

JUNE 27, 1900.

Sixth reflection; exposure 30^m; exposures with mirrors at rest at beginning and end; the first motion was +; current of 4½ amperes; under microscope the upper spectrogram corresponded to a negative, the lower to a positive direction of rotation. The difference of the readings is always expressed as lines of upper − lines of lower spectrum.

Comparison spectrum (mirrors at rest)		Spectrum with rotating mirrors	
Line 1	−0.008 rev.	λ 4461.9	+0.011 rev.
2	−0.006	4462.1	+0.007
3	−0.022	4457.6	−0.008
4	−0.016	4456.0	−0.005
		4454.7	−0.010
Mean	−0.013	4451.7	+0.003
		4448.0	+0.003
		4444.0	−0.005
		4425.6	0.000

The absolute displacement (toward red in lower spectrum) equals 0.012 rev., corresponding to a velocity of 0.75 km per sec. The mirrors had a maximum velocity of 0.55 km per sec.

JULY 1.

The comparison spectra on this plate show an unaccountably large displacement, so that at first I rejected it altogether.

If these spectra are neglected, however, and those obtained from moving mirrors are treated independently, the micrometer

thread being oriented by the dividing line between the spectra, a displacement is obtained which corresponds with sufficient accuracy to the velocity of the mirrors. The large displacement might be explained by a jar received by the spectrograph just after the first or before the last exposure.

The pitch indicated a velocity of rotation of the motor of 35 revolutions per second. Under the microscope the upper spectrum corresponded to a +, the lower to a − motion.

λ	Upper—lower spectrum
4459.4	−0.011 rev.
4457.9	−0.003
4456.0	−0.009
4451.8	−0.010
4448.0	−0.020
4444.0	+0.003
4442.5	−0.006
4425.6	−0.017
4415.7	−0.022
4408.0	−0.015
Mean	−0.011

The displacement corresponds to a velocity of 0.67, the maximum motion of the mirrors to one of 0.60 km per sec.

JULY 6.

Current = 7¼ amperes; motion at first in the negative direction; exposure 30^m. The temperature changed 0°.4 C. during the experiment. Under microscope the upper spectrum was due to −, the lower to + motion.

Comparison spectrum		Spectrum from rotating mirrors	
λ 4482.5	−0.074 rev.	λ 4462.0	−0.058 rev.
4482.5	−0.075	4456.1	−0.052
4476.2	−0.084	4451.8	−0.057
4468.7	−0.080	4436.0	−0.060
		4425.6	−0.053
Mean	−0.078 rev.	4418.0	−0.058
		4415.8	−0.064
		4407.9	−0.055
		Mean	−0.057

Absolute displacement (toward red in lower spectrum) $=0.021$ rev.; velocity $=1.28$; maximum velocity of mirrors $=0.78$ km per sec.

JULY 9.

Current $=7\frac{1}{4}$ amperes; exposure 30^m. Under microscope the upper spectrum was due to +, the lower to — motion.

	Comparison spectrum		Spectrum from rotating mirrors
1	+0.002 rev.	1	−0.007 rev.
2	+0.009	2	−0.028
3	+0.003	3	−0.009
4	+0.006	4	−0.011
5	−0.006	5	+0.009
Mean	+0.003 rev.	6	+0.002
		7	+0.006
		8	−0.0017
		9	−0.010
		10	−0.014
		Mean	−0.008 rev.

Absolute displacement (toward violet for lower spectrum) $=0.011$ rev.; corresponding velocity $=0.67$; maximum velocity of mirrors $=0.78$ km per second.

AUGUST 7.

Current $7\frac{1}{4}$ amperes; first direction of motion —; under microscope upper spectrum corresponds to — motion.

	Comparison spectrum		Spectrum from rotating mirrors
1	+0.045 rev.	1	+0.080 rev.
2	+0.062	2	+0.080
3	+0.068	3	+0.060
4	+0.064	4	+0.062
5	+0.068	5	+0.074
Mean	+0.059 rev.	6	+0.068
		7	+0.063
		Mean	+0.070 rev.

Absolute displacement (toward red for lower spectrum) $=0.011$ rev.; corresponding velocity $=0.67$; maximum velocity of mirrors $=0.78$ km per second.

AUGUST 9.

Current $7\frac{1}{4}$ amperes; exposure 30^m; first direction of motion —; under microscope the upper spectrum corresponds to — motion.

	Comparison spectrum	Spectrum from rotating mirrors	
1	+0.082 rev.	4461.8	+0.079 rev.
2	+0.071	4456.0	+0.100
3	+0.072	4451.8	+0.083
4	+0.077	4448.0	+0.086
5	+0.08	4444.0	+0.095
6	+0.075	4442.5	+0.079
7	+0.079	4437.1	+0.091
		4435.9	+0.088
Mean	+0.077 rev.	4425.6	+0.088
		4418.6	+0.087
		4417.9	+0.096
		Mean	+0.088 rev.

Absolute displacement (toward red for lower spectrum) = 0.011 rev.; velocity = 0.67; maximum velocity of mirrors = 0.78 km per second.

The velocities measured may be summarized as follows:

1900	From displacements	From rotation
June 27	0.73 km per sec.	0.46—0.55 km per sec.
July 1	0.67	0.50—0.60
July 6	1.28	0.64—0.78
July 9	0.67	0.64—0.78
Aug. 7	0.67	0.64—0.78
Aug. 9	0.67	0.64—0.78

Probable error of each velocity = ±0.17 km

These results are to be regarded as only the first experiments with the apparatus above described. Much remains in the way of its improvement, and it is especially desirable to put the wheels into a vacuum in order to avoid the resistance of the air. It is hoped that in time better results will be attained.

PULKOWA,
October 1900.

THE RADIATION OF A BLACK BODY.

By C. E. MENDENHALL and F. A. SAUNDERS.

IN the present article it is proposed to give, first, a brief review of recent work in connection with the radiation of an absolutely black body, and, second, an account of some experiments of this character carried on in the Physical Laboratory of the Johns Hopkins University. The results of the latter were largely negative; but a statement of methods and difficulties may be of service to others engaged in the same line of work.

In most cases the method of producing the "black body" has been based on Kirchhoff's discussion of the problem of radiation in a uniformly heated enclosure; a hollow body, preferably of good conducting material and having an aperture, being heated as uniformly as possible, the radiation emerging from the aperture has been taken as that of a "black body," and examined by appropriate means. Another method has been suggested and used by Paschen,[1] but it seems to be, on the whole, less satisfactory; in this case a radiating strip is put near the center of a reflecting enclosure having an aperture through which passes the radiation to be examined. In order to consider the subject to the best advantage it will be well to group the materials around the most important of the so-called "laws"—which have been obtained for the most part theoretically, and which have in turn been put to test by recent experiment. For brevity these laws will sometimes be referred to by number, corresponding to those given below, the following symbols being used:

S = total radiant energy at any absolute temperature.
T = this absolute temperature.
$E d\lambda$ = energy radiated in waves of length $< \lambda + d\lambda$ and $> \lambda$.
λ = any wave-length expressed in thousandths of a millimeter – μ.

[1] PASCHEN, *Wied. Ann.*, **60**, 1897; PASCHEN and WANNER, ASTROPHYSICAL JOURNAL, **9**, 40, 1899; **11**, 297, 1900.

λ_m = wave-length of maximum energy at temperature T.
$E_m d\lambda$ = amount of (maximum) energy at temperature T, between the limits,

$$\lambda_m \pm \frac{d\lambda}{2}.$$

A, B, C, c, α, are constants.

The following "laws" will be considered:

(I)	$S = \text{const. } T^4$.	A relation between total radiation and temperature.
(II)	$\lambda T = \text{const.}$	A relation between wave-length and temperature.
(III)	$\lambda_m T = A$.	A relation between wave-length of maximum of energy-curve and the corresponding temperature.
(IV)	$E_m T^{-5} = B$.	A relation between maximum ordinate of energy-curve and the corresponding temperature.
(V)	$E = C\lambda^{-5} e^{-\frac{c}{\lambda T}}$.	Gives distribution of energy in spectrum at any temperature; *i. e.*, is equation of energy-curve.

Equation (I), $S = \text{const. } T$,[4] expressing the well-known law of Stefan, has been subjected to experimental test recently by Lummer and Pringsheim,[1] and by Paschen.[2] The first named attacked the problem most directly and found, as can be seen from Table I, a fairly satisfactory agreement with theory.

TABLE I.

T.	373°	492°	733°	755°	790°	820°
Obs. S.	156	638	3320	3810	4440	5150
Calc. S.	143	600	3270	3700	4660	5170

T.	877°	1106°	1125°	1403°	1492°	1522°	1561°
Obs. S.	6190	16400	17700	44700	57400	60600	67800
Calc. S.	6180	17200	18500	45000	57600	62400	69100

It is to be noted here that no correction (apparently) was made for the absorption of CO_2 and H_2O vapor in the atmosphere, nor were any precautions taken to diminish this absorption. This work has since been extended to 1700° abs. in one direction and about 100° abs. in the other, and Stefan's law found to be satisfied to within a few per cent. The Stefan relation has also been deduced by Planck,[3] from the basis of the electromagnetic theory of light.

[1] LUMMER and PRINGSHEIM, *Wied. Ann.*, **63**, 395, 1897.

[2] PASCHEN, *Wied. Ann.*, **58**, 60, 1896, 1897.

[3] M. PLANCK, *Drude's Ann.*, **1**, No. 1, 1900.

The expressions (III), (IV), $\lambda_m T = A$ and $E_m T^{-5} = B$, follow at once from Wien's[1] so-called "Verschiebungsgesetz," $\lambda T =$ const. (II), and this, as originally developed, assumed the truth of Stefan's law. This "Verschiebungsgesetz" of Wien states nothing as to the distribution of energy at any one temperature, but states that this distribution must change with the temperature in such a manner that if there is any definite amount of energy corresponding to a given wave-length λ, at a temperature T, this same amount of energy will at any other temperature T_1 be emitted in waves whose length is determined by the relation $\lambda T = \lambda_1 T_1$. The expressions (III) and (IV) also are necessary consequences of (V), which, in turn, has been theoretically developed by Planck (*loc. cit.*). Thiesen[2] has objected to part of Wien's reasoning, and has deduced the relation $\lambda T =$ const. by another process.

The expressions (III) and (IV) have been very elaborately tested by experiment. Not to mention earlier work, Lummer and Pringsheim[3] obtained the following series of values for A and B:

A	B
2928	2246×10^{17}
2974	2184
2959	2176
2966	2164
2956	2166
2980	2208
2950	2166
2814	2190
2940	2188

Two other series of observations, with different arrangement of apparatus in each case, gave values for A of 2940 and 2930, and equally satisfactory constancy for B.

Paschen[4] finds, as a mean for a number of independent series

[1] W. WIEN, *Ber. d. Berl. Akad.*, **6**, 1893.

[2] THIESEN, *Verh. d. Deutsch. Phys. Ges.*, **2**, No. 5, 1900.

[3] LUMMER and PRINGSHEIM, *Verh. d. Deutsch. Phys. Ges.*, **1**, 12.

[4] PASCHEN, ASTROPHYSICAL JOURNAL, **10**, 40, 1899; **11**, 288, 1900; Paschen and Wanner, *ibid.*, **9**, 300, 1899.

of observations, extending from about 150° C. to 1300° C. and over a range of wave-lengths from 0.5 μ to 9.2 μ, the value of A to be 2907; while the maximum variation of B is 4 per cent.

In the later work of Lummer and Pringsheim the apparatus was so arranged as largely to exclude CO_2 and H_2O vapor from the atmosphere between the radiator and bolometer strip, so that the gaps due to the selective absorption of these two substances were very greatly reduced in extent and depth. On the other hand, in Paschen's later work, the bolometer strip was placed at the center of a reflecting hemisphere, in order to approach more closely to the condition of a perfect absorber. According to Kurlbaum[1] a thinly-coated bolometer strip, which appears, however, black to the eye, may absorb but 40 per cent. of the incident total radiation, so that the difference between the results of Lummer and Pringsheim and of Paschen may be partly due to the imperfect absorption of Lummer and Pringsheim's bolometer strip.

So far, then, as the expressions (III) and (IV) are concerned, which tell us nothing as to the distribution of energy in the spectrum, the predictions of theory seem to be verified to within the outstanding experimental errors, and these are being steadily reduced. Equation (V), $E = C\lambda^{-5}e^{-\frac{c}{\lambda T}}$ which concerns this distribution of energy, remains to be considered. As regards its theoretical foundation, Wien's original method is not rigorous, as has been pointed out by Lummer and Pringsheim.[2]

The most careful attempts at experimental verification have been made by Paschen[3] and by Lummer and Pringsheim.[2] Full accounts of the work of the former have been published in this JOURNAL. He finds c (see Table II for meaning of these constants) to be 14531, with a possible error of 80 from a series of experiments, while C is more variable. No systematic variation

[1] KURLBAUM, *Wied. Ann.*, **67**, 1899.

[2] LUMMER and PRINGSHEIM, *Verh. Deutsch. Phys. Ges.*, 1, 1, 1900.

[3] PASCHEN, ASTROPHYSICAL JOURNAL, **10**, 40, 1899; **11**, 288, 1900.

of either C or c is evident; he further tests (V) by plotting it logarithmically.

The theoretical and observed curves agree to within the errors of experiment, except in the region of very short wave-lengths, where the observed energy is greater than the theoretical; and precisely here, as Paschen points out, it is very difficult to avoid stray light.

On the other hand, Lummer and Pringsheim find that while the computed and observed curves (graphs of V) agree quite well in the region of wave-lengths near the maximum, particularly at lower temperatures, this agreement is not so good for the long wave-lengths, and that this disagreement increases as the temperature increases. This can be stated in another way. If the expression $E = C\lambda^{-5}\, e^{-\frac{c}{\lambda T}}$ is transformed by the introduction of logarithms we have $\log E = \log C - \frac{c}{\lambda T} \log e - 5 \log \lambda$, and if $\log E$ is plotted against $\frac{1}{T}$ we have the so-called "isochromatic" curve, evidently a straight line. This may be compared with the curve plotted for corresponding values of E and T, observed at a fixed point in the spectrum (λ constant). Evidently the slope of this line is proportional to c, while from the constant term can be obtained the value of C. According to Lummer and Pringsheim's observations the observed isochromatic is convex toward the $\frac{1}{T}$ axis, and the values for C and c obtained from these isochromatic curves increase systematically as the temperature rises. These investigations of Lummer and Pringsheim are still in progress. Thiesen has, however, found, from a recalculation of the results of Lummer and Pringsheim, that a modification of V, by changing the coefficient of λ from -5 to -4.5, would completely satisfy their observations. The law thus modified would be satisfied by the observations of Beckmann,[1] and is further strengthened by some recent work of Lummer and Pringsheim.[2]

[1] BECKMANN, *Inaug. Diss.*, Tübingen, 1898.

[2] Referred to by THIESEN, *Verh. d. Deutsch. Phys. Ges.*, **2**, 5, 1900.

As regards the variation of these constants, Rubens[1] has discussed the results of Beckmann, who used a hollow "black body" as source of radiation, and produced a more or less perfect isolation of certain wave-lengths by repeated reflection from fluor spar, and thus determined an approximate "isochromatic" curve in a spectral region not heretofore studied in this connection, viz., for a mean wave-length of about $28\,\mu$. In order that the Wien formula (V) should represent Beckmann's work (as recalculated by Rubens) it is necessary that c should have the value 26000. On account of the method used, this result ought not, perhaps, to be considered conclusive. However, Rubens points out that the change in the ordinate of the energy curve at $\lambda = 25\,\mu$ produced by a change in c from 26000 to 14500 would be (at 2000° C.) $\frac{8}{10000}$ of the maximum ordinate; so that it would be difficult to detect such a change in c by study of the energy curves.

The present knowledge respecting these various laws of radiation can perhaps best be summed up in the following table:

TABLE II.

(I) $S = \int E d\lambda = \text{const. } T^4$.	First given by Stefan.[2] Thermodynamically deduced by Boltzmann[3] with certain assumptions. Experimentally tested by Paschen[4] and more especially by Lummer and Pringsheim.[5] Deduced by Planck from electro-magnetic theory, involving electro-magnetic definition of entropy and temperature.

[1] RUBENS, *Wied. Ann.*, **69**, 1899.

[2] STEFAN, *Sitzber. d. k. Gesellsch. zu Wien*, **79**, 1879.

[3] BOLTZMANN, *Wied. Ann.*, **22**, 1884.

[4] PASCHEN, *Wied. Ann.*, **58**, 1896; **60**, 1897.

[5] LUMMER and PRINGSHEIM, *Wied. Ann.*, **63**, 1897.

(II) $\lambda T = \text{const.}$
from which follow:

(III) $\lambda_m T = A$.

(IV) $E_m T^{-5} = B$.

Developed theoretically by Wien,[1] assuming Stefan's law (I). Tested experimentally by Paschen,[2] Lummer and Pringsheim,[3] Paschen and Wanner,[4] and found to hold with increasing accuracy as experimental methods are improved. Outstanding difference between Paschen and Lummer and Pringsheim of about 1 per cent. in value of A; B not so good. (II) Theoretically developed by Thiesen,[5] who questions the rigor of Wien's original method, and by Planck[6] from an electro-magnetic basis.

(V) $E = C\lambda^{-5} e^{-\frac{c}{\lambda T}}$.

(III) and (IV) followed by differentiation from (V).

(VI) $E = C\lambda^{-a} e^{-\frac{c}{\lambda T}}$.

Theoretically deduced by Wien[7] —but with rather arbitrary assumptions and not altogether rigorous reasoning.[8] An expression of the same form (VI) was given by Paschen as best representing the energy curves of various radiating surfaces. Tested by Lummer and Pringsheim,[3] who found systematic variations in C and c with temperature; also by Paschen and Wanner[4] and Paschen,[9] who finds no systematic variation of the constants and a quite satisfactory agreement of the observed and computed curves. Also deduced theoretically by Planck.

(III) and (IV) are necessary, but not sufficient conditions for the truth of (V).

[1] Wien, *Ber. d. Berl. Akad.*, **6**, 1893; *Wied. Ann.*, **52**, 1894.

[2] Paschen (*loc. cit.*) and Astrophysical Journal, June 1899, May 1900.

[3] Lummer and Pringsheim, *Verh. d. Deutsch. Phys. Ges.*, **1**, 1 and 12, 1899.

[4] Paschen and Wanner, *Ber. d. Berl. Akad.*, January 1899.

[5] Thiesen, *Verh. d. Deutsch. Phys. Ges.*, **2**, 5, 1900.

[6] Planck, *Drude's Annalen*, **1**, 1900; **4**, 1900.

[7] Wien, *Wied. Ann.*, **58**, 662, 1896.

[8] Lummer and Pringsheim, *Verh. d. Deutsch. Phys. Ges.*, **1**, 1, 1899.

[9] Paschen, *Wied. Ann.*, **60**, 1897.

Here

S = total energy of radiation at any absolute temperature.

T = total energy of radiation at this absolute temperature.

$E d\lambda$ = energy radiated in waves of length $< \lambda + d\lambda$ and $> \lambda$.

λ = any wave-length in μ, (0.001 mm.)

λ_m = wave-length of maximum energy at temperature T.

$E_m d\lambda$ = amount of maximum energy at temperature T, between the limits

$$\lambda_m \pm \frac{d\lambda}{2} \; .$$

A, B, C, c, a, are constants.

The following work on the radiation of an "absolutely black body" was begun by Mr. C. E. Mendenhall in conjunction with Dr. H. F. Reid, the part relating to temperatures above 500° C. being carried out by him, while that relating to temperatures below 500° C. was carried out by Mr. F. A. Saunders. The work was done in the Physical Laboratory of the Johns Hopkins University, Baltimore.

We shall first consider the work at temperatures above 500° C., then point out the differences in procedure adopted for the low temperature curves, and give the corresponding results.

The method of realizing the "black body," based on Kirchhoff's theoretical investigation of the radiation inside a uniformly heated inclosure, had been suggested by Dr. Reid in this JOURNAL,[1] and independently by Wien and Lummer.[2]

Our black body was either a cast-iron or copper cylinder, about 8 cm in diameter and 12 to 18 cm high, with a slit in the side through which passed the radiation to be examined; a furnace, or, for the lower temperatures, appropriate baths, served for the heating.

The spectrometer was practically a reproduction of Langley's early one, used at Allegheny.

The available rock salt consisted of a 60° prism, having faces about 5 cm × 7 cm, and a lens, about 10 cm diameter and 40 cm focus, with Brashear surfaces — the material for which had been kindly loaned by Professor Langley.

[1] H. F. REID, ASTROPHYSICAL JOURNAL, **2**, 160, 1895.

[2] W. WIEN and O. LUMMER, *Wied. Ann.*, **56**, 1895.

In order to increase the sensitiveness of the bolometer, two diagonal arms of the Wheatstone bridge quadrilateral were exposed to radiation. Theoretically this should be better than a single strip of the same width in the ratio $\sqrt{2}$ to 1. Considerations of freedom from "drift" with general changes in temperature made it desirable that the balancing arms should be as nearly like the exposed strips as possible, and similarly situated. All four arms were accordingly made of similar strips of annealed platinum foil, and mounted in the same cell. Balancing was accomplished by moving the galvanometer terminals independently along two copper wires; one, being comparatively fine (No. 24 about), gave rough adjustment, while the other (No. 12), gave fine adjustment.

The spectrometer, balancing bridge, etc., were inclosed in a double-walled box — for the purpose not only of protecting the bolometer and its appendages from temperature changes, but also of protecting the rock salt from moisture. No attempt was made to exclude or remove CO_2, nor water vapor, except in so far as was needed for the protection of the rock salt. The battery consisted of a number of Edison-Lalande cells, connected in multiple, and, on the advice of Mr. C. G. Abbot, of the Smithsonian Institution, carefully protected, as were the main leads, also, from temperature changes. The galvanometer was of the Thomson 4-coil pattern, of low resistance (about 4 ohms), and with the needle used in the first part of the work gave 1 mm deflection at 1 m with about 2.5×10^{-10} amp., with a complete period of 10 sec., though only rarely was this maximum sensibility needed.

The bolometer showed a very satisfactory freedom from "drift" and from disturbance in general. The galvanometer, however, located, as it was, in the midst of a city, was, with the best needle-system which we were able to produce, and with a quadruple iron magnetic shield, quite unusable in the day time, so that all observations had to be carried on at night. For temperatures above 500° the black body was heated in a furnace of fire-clay and the temperatures were determined by the use of

several platinum, platinum-iridium thermo-couples, according to the potentiometer method, much as outlined by Barus. These were calibrated by the use of a number of standard melting and boiling temperatures, viz., water, napthalin, mercury, potassium chloride and gold. The higher temperature determinations were perhaps in error by 5° or 10°. Temperatures were measured at four points, two at the top of the cylinder (black body), and two at the bottom. With the furnace method of heating the black body, as we used it, differences of temperature of from 10° to 20° were usually found between some of these four points.

PART I.

With the above apparatus observations of the distribution of energy in the black body spectrum were taken at many temperatures between 500° C. and 1100° C.; and a few sets of observations of energy at various (fixed) points in the spectrum while the temperature varied — giving data for the so-called isochromatic curves. When the observations were used with the corresponding observation of minimum deviation to plot energy curves, the characteristic absorption bands of CO_2 and H_2O vapor were very marked. These curves were then put in the normal form by changing from minimum deviation to wavelength, using the dispersion curve of rock salt found by Rubens,[1] and by Rubens and Trowbridge.[2] The corresponding change in the ordinates of these curves, viz., multiplication by $\frac{d\lambda}{d\delta}$, was made—also corrections for impurity of spectrum, according to Runge,[3] and for variation in sensibility of apparatus.

It was at first attempted to allow for the absorption bands of H_2O and CO_2 in the usual way by "bridging over" the gaps with a free-hand curve. Upon comparing these curves, however, it was concluded that the amount of absorption had, over part

[1] RUBENS, *Wied. Ann.*, **54**, 436, 1895.

[2] RUBENS and TROWBRIDGE, *Am. Jour. Sci.*, January 1898.

[3] RUNGE (Paschen) *Wied. Ann.*, **60**, 1897, and Schlömilch's *Zeit. fur Math. u. Phys.*, **43**, 1897.

of the curve, been greatly underestimated. This made the entire middle portion of the curves uncertain; especially it made the wave-length of maximum energy very difficult to determine, and hence made it impossible to test accurately equation (III). In fact, by properly bridging over the absorption gaps, the curves can be made to satisfy (III) as exactly as may be desired. Paschen[1] has stated that the expression

$$\lambda_m = \frac{(\log \lambda_2 - \log \lambda_1)\, \lambda_2 \lambda_1}{(\lambda_2 - \lambda_1) \log \epsilon},$$

where (λ_1, λ_2) are any two wave-lengths on opposite sides of the maximum corresponding to equal energies of radiation, serves to give consistent values for λ_m, and it has accordingly been used in connection with these curves. The wave-lengths (λ_1, λ_2) were taken at points where the absorption was as small as possible, and for each of seven curves several values of λ_m were calculated; these values agreed usually to about 0.1 μ. The resulting values of $\lambda_m T$ are as follows:

$T, C°$	λ_m	$\lambda_m T$
570	3.34	2815
704	2.72	2657
771	2.53	2641
837	3.36	2619
896	2.20	2571
944	2.20	2611
1030	2.00	2586

With the exception of the first one, the numbers in the last column are as nearly constant as could be expected considering the possible errors of measurement—but the mean value differs by nearly 300 from Paschen's mean value 2907, or Lummer and Pringsheim's 2930. This could be accounted for by imperfect "blackness" of our radiator, but this seems a rather improbable explanation considering the size and form of our enclosure. It is perhaps more probable that the heavy absorption on the descending side of our curves has led to an apparent shifting of all the λ_m toward the short wave-lengths.

[1] PASCHEN, *Wied. Ann.*, **50**, 409.

As far as (I) is concerned our method is at best a poor one — analyzing the radiation only to integrate the energy-curve afterwards; with the absorption as large a part of the total energy as the curves would indicate, an attempt to confirm (I) becomes still less fruitful.

As for (IV) E_m is rendered uncertain by the bands above referred to — but not to the same extent as λ_m; for the entire change in λ_m through the temperature range used in the high temperature work is but about 1.2 μ; and an examination of the curves shows that the uncertainties are a large part of this.

The causes of this extremely strong absorption undoubtedly lay in the use of a furnace to heat the black body, which became filled with the products of combustion, notably CO_2 and H_2O. That no more elaborate means to prevent this were taken was due to the conclusion, drawn from a comparison of some of the final curves roughly plotted, with some curves previously taken with slightly different arrangements — that the amount of furnace gas in the black body was not sufficient to produce extraordinary absorption. This conclusion was evidently in error.

The following table gives the values of (B) for seven curves:

°C	T	$E_m T^{-5} \times$ const.
1020	1293	59
914	1187	65
896	1169	61
837	1110	50
771	1044	55
704	977	51
570	843	54

As to Stefan's law (I), S can be approximately determined from the area of the various curves as finally corrected. From these we obtain the following table of values of $\frac{S}{[T^4 - T_1^4]}$, where T_1 is the absolute temperature of the shutter used to exclude radiation.

T	$\frac{S}{[T^4 - T_1^4]} \times$ const.
1293	493
1187	528
1169	457
1110	436
1044	470
977	458
843	433

Here also there is very unsatisfactory constancy. The five lower temperature curves agree fairly well among themselves, but we think it probable that the absorption has not been completely allowed for in these curves. The error in the 1187° curve seems to be rather larger than could be accounted for by an error in the absorption correction alone; unless, as suggested above, the other curves have been undercorrected. If this is the case, then the coincidence of an error of 8° in the estimation of temperature (for the 1187° curve) with an easily allowable overestimation of the absorption correction would account for the discrepancy.

Part II.

On account of the extremely small amount of radiation with which one has to deal in measuring the energy in the spectrum of a radiating body at comparatively low temperatures, it was absolutely necessary to have a more sensitive needle system in the galvanometer for this part of the work, and accordingly a series of experiments was undertaken to determine what form of system would be most efficient. The vertical needle system of Weiss having proved inadequate when the highest sensibility was required, the ordinary form of system was used and a number of modifications in it were tried, with results which it may be of interest to give. It seemed obvious from the work of Paschen and others that the lighter the system was, other things being equal, the more sensitive it was, and also that the greater the proportion of the weights of the magnets to the total weight, the greater the sensitiveness. Starting from these facts, fourteen different systems were made and tested. The magnets used throughout were made from watch and clock-spring material and were

tempered, magnetized and boiled before being mounted. A great many magnets were made at the beginning and from these the best were selected by observing their activity when laid upon a glass plate and tapped in a vertical magnetic field; only the best were used. The sensitiveness of each system was found by mounting it on a fine quartz fiber in the galvanometer and observing the deflection produced by a measured current. The figures given for this result are the currents in amperes which produced a deflection of 1 mm on a scale 1 meter distant when the complete period of the system was ten seconds.

Each system was built on a very fine glass rod and furnished with a minute copper wire loop at its upper end, by means of which it could be hung on a corresponding hook on the lower end of the fiber. The mirrors used were fragments of the finest microscope cover-glasses obtainable, silvered, and cut into pieces roughly circular in shape with an area of about 1.5 sq. mm. Their weights varied from 0.4 to 0.7 mg.

A study of the results obtained brought out the following conclusions, which, of course, apply exactly only to a galvanometer whose "free space" is circular and of the same diameter as ours (3 mm):

1. It is unwise to make the magnets shorter than 1 mm.
2. It is unwise to make the system as deep (measured along the stem) as the diameter of the free space.
3. It is somewhat disadvantageous to make the magnets themselves as long as the free space is wide; such systems are also very troublesome to use.
4. There is a slight disadvantage in making the magnets of material thinner than 0.05 mm.

One system was finally chosen as the best of all, and the systems used in the subsequent work were made after the same pattern. Each group in the system consisted of three magnets, two of which were 1.6 mm long while the third was 2.3 mm long; the width of each magnet was 0.2 mm and its thickness 0.05 mm. Each group was spaced along the stem so as to cover about 1.5 mm. The total weight was 2.5 mg; of which 1.3 mg was of steel, 0.6 mg of mirror and 0.6 mg of glass and shellac.

An effort was next made to bring a number of these systems to a highly astatic condition. This, of course, requires that the magnets shall lie in the same plane, or in parallel planes, and that the upper and lower sets shall be equal in magnetic strength but opposite. As this occupied an unexpectedly long time, it may perhaps be well to give an account of the difficulties encountered in this apparently simple operation. In the first systems the magnets were all fastened on the same side of the glass rod and the mirror was on the opposite one. The magnets of the systems being fastened in place while lying on a piece of plate glass they were nearly all in the same plane, and the fine adjustment was made by loosening one of the magnets a little by heating and turning it through a small angle. For a considerable time no consistent results were obtained, owing to the proximity of a magnetized steam-pipe which had an unexpectedly great effect on the uniformity of the Earth's magnetic field in the place in which the systems were kept. Having moved the systems to a place where the field was uniform, we once more adjusted them by turning one magnet of one set so that the plane of the equivalent magnet was parallel to that of the other set. This process, since it involved heating, usually resulted in a slight weakening of the magnet turned, which was corrected by bringing near it a powerful permanent magnet. Any system the planes of whose magnets are not parallel should, when the magnetic strength of the two sets are made equal, stand with its magnets pointing east and west, and its period should be longer the nearer its magnet sets are to being in the same plane. This was found to be the case in about one system only out of ten. The others as they approached an astatic condition reached a condition when they would oscillate about *two* positions of equilibrium, usually in the two directions northwest to southeast and northeast to southwest. When this was the case, there was no means of finding, by its positions of equilibrium or by its period of oscillation, which of the two magnet sets was the stronger, nor in what direction one of them must be turned in order to bring the sets more nearly into the

same plane. It was therefore impossible to proceed with the astaticizing at all.

This anomalous condition was in no way due to torsion in the supporting fiber, for a complete revolution of this produced no more than five degrees change in the natural position of the system, and six whole turns were necessary to force the system to revolve. The fibers were from 15 to 25 cm in length and not more than 0.0025 mm in diameter, and usually somewhat less. They were made by the blowing process invented by Boys,[1] in which the oxy-hydrogen flame melts and at the same time blows out a minute fragment of quartz into a long fiber, a process which proved extremely efficient and simple after a little practice had been obtained with it.

Each fiber was hung in a glass tube so that its lower end projected beyond the tube into a small wooden box whose front was closed by a glass plate fastened by a bit of wax. It was found that the act of detaching this glass plate charged it quite highly with electricity, and it was hoped that the action between this charge and the charge induced on the magnets might account for the existence of two positions of equilibrium. That it did not, however, was amply shown by the use of a brass plate instead of the glass one, which was carefully discharged whenever handled, by passing it through a flame. The systems behaved just as before.

A lack of symmetry in the system itself was next suggested as a possible explanation, though by no means, at first sight at least, a very rational one. Accordingly, several systems were constructed and astaticized in which the following precautions were taken: the weights of the two sets of magnets were made equal to within 0.1 mg; the weight of the magnets fastened on one side of the glass stem was made equal to the weight of the magnets and mirror on the other side of the stem, to the same degree of approximation (it was not possible to do this more accurately, as the total weight of the parts was not often more than 1 mg and the weighings were not certain to less than

[1] V. C. Boys, *London Electrician*, December 11, 1896.

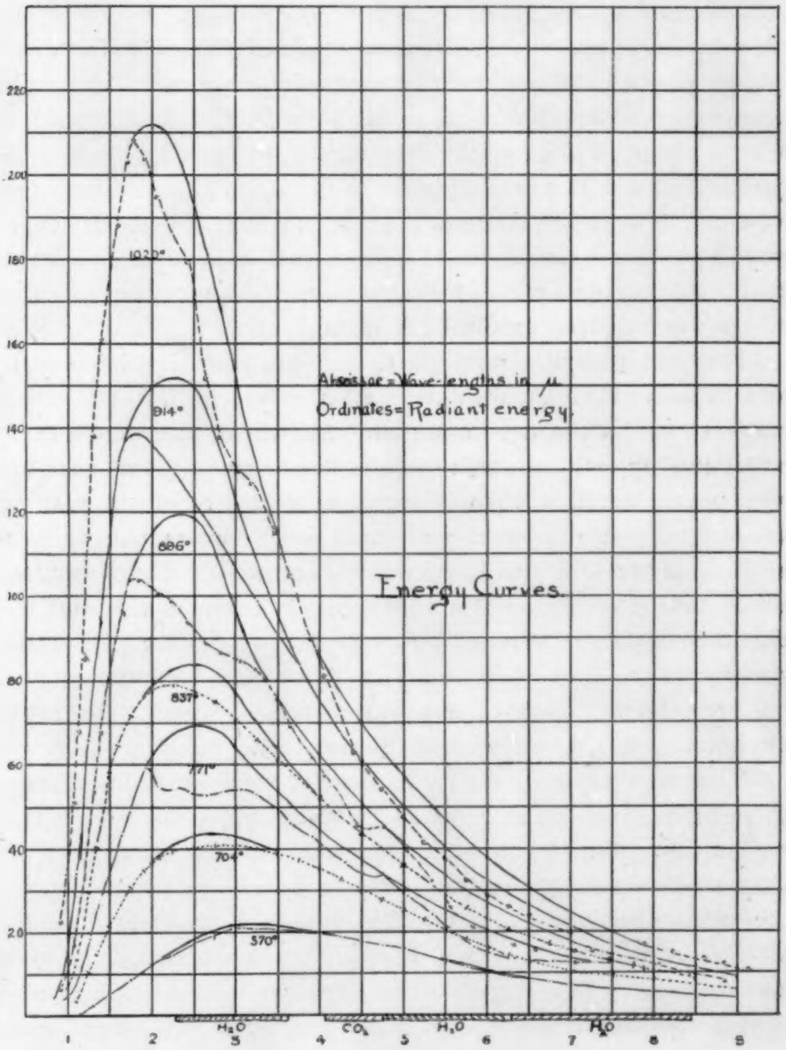


FIG. 1.

Broken lines, uncorrected or partially corrected curves.
Full lines, corrected curves, drawn to fit relation λ_m T = const.
Absorption bands indicated at bottom.

0.1 mg or to 10 per cent.); the greatest care was taken to have the glass stem fastened at the middle points of the magnets, and the mirror was so adjusted as to make the oscillations of the whole system aperiodic when it was supported horizontally so as to be free to turn about the stem as axis; and, finally, the loop of wire at the upper end of the system was dispensed with and the quartz fiber was fastened directly to the stem. The systems that were produced after all these precautions were taken were in no way better than those made before, and as no further changes could be thought of which would prevent this anomalous behavior all the systems that exhibited it were discarded.

It would be quite possible to account for this difficulty by remembering that an absolutely astatic system, *i. e.*, one whose magnets were all in the same plane and whose sets were exactly equal and opposite in strength when they were in an east and west plane, would be thrown out of astaticism when the plane of the magnets became north and south by the magnetism induced in the magnets by the Earth's field, and would hence oscillate about two positions of equilibrium. It seemed very unlikely that these systems were so nearly perfect that this explanation could apply to them, particularly as the induced magnetism must be very feeble in hardened watch-spring steel. No other explanation has, however, been thought of.

The best systems finally chosen, three or four in all, were then brought to a fairly astatic condition, as shown by their period of oscillation (complete period six or seven seconds), and they were then examined from day to day, being as far as possible undisturbed in the intervals, to see how their condition altered with the time. This alteration proved to be very great, and was apparently as great in the case of systems composed of boiled magnets as it was with some constructed of "raw" material. All of them at first lost their astaticism rapidly, but, after readjusting them once or twice a day, at the end of two weeks we obtained one which held a period of about five seconds for the ensuing two weeks without appreciable alteration. This system was therefore mounted in the galvanometer and was used

in the subsequent work. It had a sensitiveness of about 1×10^{-10} amp. At the close of the investigation, however, after three months use, it was found to have a period of about three seconds, indicating a considerable fall in astaticism.

Seven curves in all were obtained, all of them duplicated in important regions, namely, at the temperatures 100°, 175°, 243°, 313°, 399°, 503°, and 578° C. Of these curves the four taken at the lowest temperatures are drawn in Fig. 2, and reveal at once the presence of the absorption bands due to the presence of carbon dioxide and of water vapor in the air, which are to be found in all the curves taken. The full lines with which the observed curves in part coincide indicate the curves filled in according to a method explained below. It was hoped that five or six vessels containing concentrated sulphuric acid, which were put inside the spectrometer box, would keep the air inside reasonably dry. They did keep the rock-salt surfaces from being fogged, but did not prevent the water-vapor bands from being prominent in every curve. With the spectrum as impure as it was, these bands in some places overlap and affect the curves continuously for a considerable distance. A few points in the spectrum seemed, however, to be free from absorption, and the observations at these points only were used.

Paschen[1] has found that if Wien's law be true, and if the radiation curves be plotted, not as is usually done, with wave-length and intensity as coördinates, but with the logarithms of these quantities instead, then the curves have the property of congruency. By this is meant that any one curve is an exact copy of any other, but shifted, unaltered in shape, to another part of the diagram. Now, since the maximum energy occurs at different wave-lengths as the temperature is changed, it is evident that any region of absorption will fall upon a different part of each curve, and hence if the curves are congruent as above explained, the points cut out of any curve by absorption can be supplied from another curve by merely laying one above the other. In this way, by the use of curves taken at different temperatures,

[1] F. PASCHEN, *Wied. Ann.*, **60**, 1897.

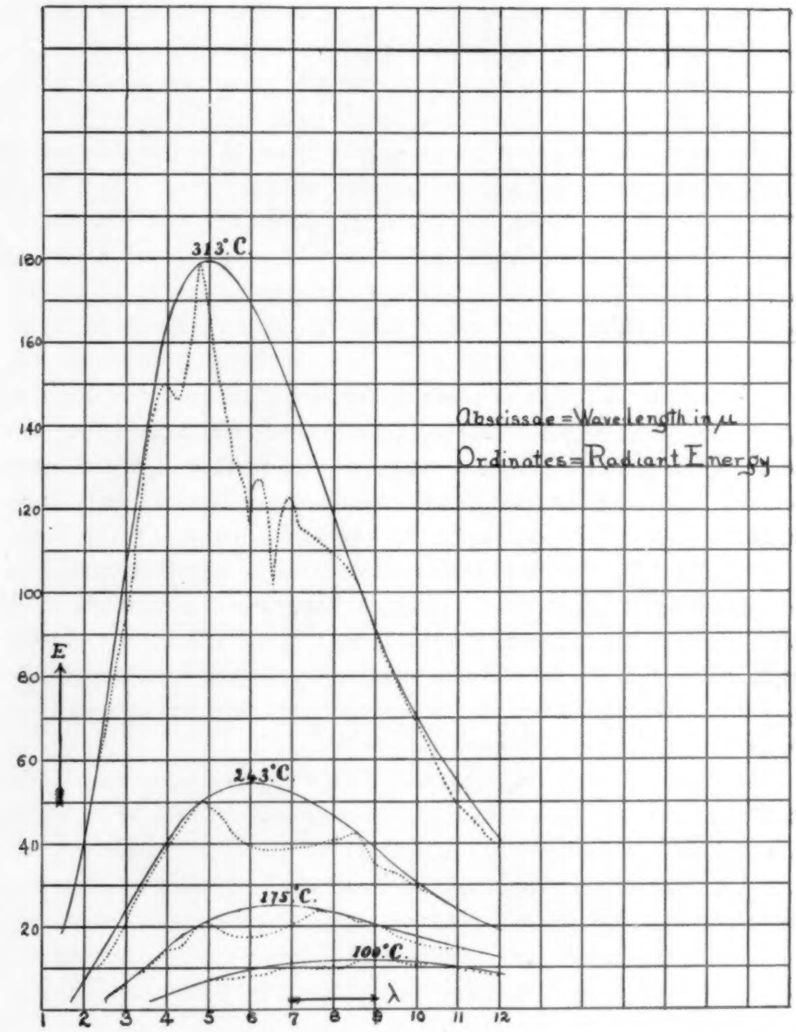


FIG. 2.

the entire curve can be constructed from a few points. This has been the method used. Our curves when plotted logarithmically are roughly congruent, though not accurately so, as there seems to be a slight change in form as the temperature changes. The curve is roughly of an inverted U shape, and our curves show a

slight tendency towards a widening of the U as the temperature increases. The mean curve was, however, taken, as the deviations did not seem too large, and by means of this the missing points in the observed curves were supplied.

Four tests were applied to this set of curves. Law I was first tested, with the results shown by the following tables:

T	λ_m	$\lambda_m T$
373°	8.30	3090
448	6.50	2910
516	5.90	3040
586	4.90	2870
672	4.37	2940
776	3.81	2950
851	3.24	2760
	Average	2940

These figures are somewhat irregular, but the determination of the maximum of the energy curve is an extremely uncertain thing when the curves are as much cut up by absorption bands as these were. The average value of this constant, 2940, is somewhat higher than that found by Paschen, and there is a slight tendency shown in the series of values towards an increase in the value of the constant with decrease of temperature. The marked difference between this value and that found in the earlier part of this work may perhaps point to the same result. The expression (IV) was next tested. This set of observations fails entirely to conform with this law. The maximum energy varies nearly as the sixth power of the absolute temperature.

The following relation should also hold:

$$\frac{E}{E_m} = \left\{ \frac{\lambda_m}{\lambda} \cdot e^{\frac{\lambda - \lambda_m}{\lambda}} \right\}^a,$$

where E is the intensity at wave-length λ, and E_m is the maximum intensity. This relation was found experimentally by Paschen, and according to Wien's law the constant a should be 5. This constant was calculated from a great number of points on all the curves, and the average value obtained was slightly less than 5, but very near it.

Finally, Wien's expression (V) for the energy at any wave-length, gives, on integration with respect to λ from O to λ,

$$\int_0^\lambda C\lambda^{-5}\epsilon^{-\frac{c}{\lambda T}} \cdot d\lambda = C\left(\frac{T}{c}\right)^4 \cdot \lambda^{-3} \cdot \epsilon^{-\frac{c}{\lambda T}} \cdot \left\{\left(\frac{c}{T}\right)^3 + 3\left(\frac{c}{T}\right)^2\lambda + 6\left(\frac{c}{T}\right)\lambda^2 + 6\lambda^3\right\}.$$

This enables us to compare the areas of the curves up to a certain wave-length with those required by the formula. If we knew the entire curve we could simply integrate it and then its area, representing as it does the total radiation, should follow Stefan's law, *i. e.*, should be proportional to the fourth power of the absolute temperature. It is not possible, however, to do this on account of the large part of the curve lying in the region of the longer wave-lengths, which is influenced by the absorption of the prism, etc. This circuitous method was tried on our curves, and since Wien's law is based on Stefan's, should lead to the same result. This was not found to be the case here, our areas being nearly proportional to the seventh power of the absolute temperature; but this method of obtaining the total radiation is too indirect to lead to accurate results.

The conclusions to be drawn from our results are, as before stated, rather negative in character. It is evident that some of the deductions from Wien's law are satisfied, while others are not; but no results of sufficient accuracy can be obtained without taking excessive precautions in regard to the "blackness" of both radiator and bolometer strip, and in excluding from the air about the apparatus all traces of carbon dioxide and water vapor.

It is interesting to note that if law III holds, the maximum of the radiation-curve of a body at the temperature of the boiling of liquid air under atmospheric pressure should lie at about 30μ, and would therefore be beyond the reach of any rock-salt dispersion apparatus. An effort was made to test this by cooling the black body with liquid air, but at the last it proved impossible to obtain enough for our purposes. With the small

quantity that we had, however, the body was cooled to about —90° C., and at this point caused the greatest deflection at about 10 μ, which is roughly where the maximum should lie, according to law III. The deflection was also of the order to be expected, though the working conditions at the time were not good enough to allow any accurate measurements to be taken.

The writers wish to express their sincere indebtedness to Dr. H. F. Reid for his continued interest and valuable advice, and to Professors Rowland and Ames, not only for their kind supervision, but for the generosity with which they placed all the facilities of the laboratory at our disposal.

NOTE.—Mr. H. C. Dickinson of Williams College has suggested that the two positions of equilibrium of the delicate galvanometer needles, above referred to, may be due to the disturbance of the co-planarity of the two systems of magnets, by the couples between these systems and the magnetic field, opposed by the torsional rigidity of the connecting glass rod. This assumes that the two groups of magnets are initially very closely in the same plane—and of very nearly equal moments. Some experiments have been performed which indicated that a change of several minutes might be expected in the angular relation of the two groups, and this seems sufficient to account for the observed phenomena. C. E. M.

November 1900.

EXAMINATION OF *PLEIADES* AND *EROS* PLATES TAKEN WITH THE CROSSLEY REFLECTOR OF THE LICK OBSERVATORY.[1]

In reply to a letter received September 21, 1900, from the Acting Director of the Lick Observatory, Professor W. W. Campbell, requesting the aid of the Columbia University Observatory in determining the accuracy of the photographs of *Eros* to be taken with the Crossley reflector,[2] Professor Rees stated that the Columbia Observatory staff would make very gladly the measurements required.

Professor Campbell was asked to send three plates of the *Pleiades* taken on the same night or different nights, having the center of each plate near the greatest number of stars. In the meantime a plate of *Eros* was taken on September 19 with the Crossley reflector and forwarded to the Columbia Observatory. The *Pleiades* plates were obtained on September 27 and received October 8. The plates were placed at once in charge of Professor Jacoby.

Dr. S. A. Mitchell and Miss F. E. Harpham made the measurements and in the reductions were assisted by the Observatory computing force. The results of the measurements and reductions are given below in the report from Professor Jacoby.

REPORT ON THE MEASUREMENT AND REDUCTION OF THREE PLEIADES PHOTOGRAPHS MADE WITH THE CROSSLEY REFLECTOR OF THE LICK OBSERVATORY.

The following pages contain a detailed account of the measurement and reduction of three photographs of the *Pleiades*, made with the Crossley reflector of the Lick Observatory.

[1] Communicated by Professor J. K. Rees, Director of Columbia University Observatory, New York City.

[2] *Note by W. W. C.*—The Crossley reflector is in charge of Assistant Astronomer C. D. Perrine. He is assisted by Mr. H. K. Palmer, Fellow in Astronomy. The plates were secured by them.

The data relative to the exposure of the negatives are contained in Table I.

TABLE I.

Plate	Date	Exposure	Mt. H. Sid. Time (Middle of exposure)	Barometer	Att. Therm.	Ext. Therm.
	1900					
1.....	Sept. 27	30 sec.	4^h 2^m $50^s.0$	26.127	57°.0	56°.1
2.....	" 28	10 sec.	4 17 39 .0	26.137	61.0	58.3
3.....	" 27	5 sec.	0 47 5 .5	26.127	57.0	57.0

The plates were measured in rectangular coördinates by two observers and reduced according to Jacoby's method as given in Dr. Schlesinger's paper on the *Praesepe* Group.[1]

Table II contains the results of the individual measures in millimeters, and the differences between the two observers reduced to seconds of arc. Ten stars were measured on each plate, the central star numbered 17 being observed twice. The numbers in the first column are from Jacoby's catalogue of the *Pleiades*.[2]

TABLE II.

PLATE I. x MEASUREMENTS.

Star	x direct				x reversed			
	Line of Scale	½ m or Scale *minus* Star		Mitchell *minus* Harpham	Line of Scale	½ m or Scale *minus* Star		Mitchell *minus* Harpham
		Mitchell	Harpham			Mitchell	Harpham	
1	111	0.0470	0.0592	−0″.47	8	0.5750	0.5840	−0″.35
5	96	0.0548	0.0495	+ .20	23	0.5808	0.5858	− .19
16	71	0.7915	0.7975	− .23	47	0.8405	0.8410	− .02
17	70	0.5398	0.5455	− .22	49	0.0855	0.0908	− .20
17	70	0.5455	0.5370	+ .33	49	0.0890	0.0905	− .06
19	69	0.2488	0.2460	+ .11	50	0.3892	0.3902	− .04
20	66	0.3750	0.3755	− .02	53	0.2598	0.2655	− .22
22	65	0.4765	0.4702	+ .24	54	0.1530	0.1628	− .38
24	61	0.7955	0.7970	− .06	57	0.8368	0.8410	− .16
25	57	0.7328	0.7350	− .08	61	0.9005	0.9005	.00
34	37	0.8740	0.8935	− .75	81	0.7498	0.7390	+ .42

[1] "The *Praesepe* Group: Measurement and Reduction of the Rutherfurd Photographs," by FRANK SCHLESINGER, *Annals N. Y. Acad. Sci.*, **10**, 189–286, 1898, or *Contrib. Obsy. Columbia Univ.*, No. 15.

[2] *Annals N. Y. Acad. Sci.*, **6**, 323, or *Contrib. Obsy. Col. Univ.*, No. 3.

PLATE II. *x* MEASUREMENTS.

Star	*x* direct				*x* reversed			
	Line of Scale	½ *m* or Scale *minus* Star		Mitchell *minus* Harpham	Line of Scale	½ *m* or Scale *minus* Star		Mitchell *minus* Harpham
		Mitchell	Harpham			Mitchell	Harpham	
1	115	0.3722	0.3815	−0.″36	4	0.2488	0.2618	−0.″50
5	100	0.3765	0.3852	− .34	19	0.2520	0.2560	− .15
16	76	0.1235	0.1165	+ .27	43	0.5070	0.5035	+ .14
17	74	0.8748	0.8865	− .45	44	0.7465	0.7498	− .13
17	74	0.8800	0.8855	− .21	44	0.7462	0.7480	− .07
19	73	0.5895	0.5895	.00	46	0.0402	0.0390	+ .05
20	70	0.7115	0.7225	− .42	48	0.9132	0.9088	+ .17
22	69	0.8332	0.8438	− .41	49	0.7935	0.7878	+ .22
24	66	0.1075	0.1120	− .17	53	0.5090	0.5118	− .11
25	62	0.0915	0.0990	− .29	57	0.5365	0.5320	+ .17
34	42	0.2602	0.2700	− .38	77	0.3670	0.3698	− .11

PLATE III. *x* MEASUREMENTS.

Star	*x* direct				*x* reversed			
	Line of Scale	½ *m* or Scale *minus* Star		Mitchell *minus* Harpham	Line of Scale	½ *m* or Scale *minus* Star		Mitchell *minus* Harpham
		Mitchell	Harpham			Mitchell	Harpham	
1	119	0.1038	0.0975	+0.″21	0	0.5135	0.5272	−0.″53
5	104	0.1158	0.1090	+ .26	15	0.5110	0.5170	− .23
16	79	0.8710	0.8702	+ .03	39	0.7638	0.7620	+ .07
17	78	0.6218	0.6158	+ .23	41	0.0098	0.0145	− .18
17	78	0.6208	0.6220	− .05	41	0.0088	0.0060	+ .11
19	77	0.3275	0.3258	+ .07	42	0.3155	0.3078	+ .30
20	74	0.4670	0.4610	+ .23	45	0.1670	0.1788	− .46
22	73	0.5668	0.5495	+ .67	46	0.0765	0.0790	− .10
24	69	0.8615	0.8680	− .25	49	0.7658	0.7702	− .17
25	65	0.8248	0.8300	− .20	53	0.8062	0.8038	+ .09
34	46	0.0098	0.0060	+ .15	73	0.6262	0.6345	− .32

PLATE I. *y* MEASUREMENTS

Star	*y* direct				*y* reversed			
	Line of Scale	½ *m* or Scale *minus* Star		Mitchell *minus* Harpham	Line of Scale	½ *m* or Scale *minus* Star		Mitchell *minus* Harpham
		Mitchell	Harpham			Mitchell	Harpham	
1	63	0.7568	0.7670	−0.″39	55	0.8630	0.8588	+0.″16
5	65	0.7032	0.7028	+ .02	53	0.9278	0.9235	+ .17
16	72	0.2210	0.2228	− .07	47	0.4070	0.4080	− .04
17	60	0.3382	0.3442	− .23	59	0.2932	0.2948	− .06
17	60	0.3368	0.3412	− .17	59	0.2912	0.2920	− .03
19	55	0.9748	0.9752	− .02	63	0.6495	0.6585	− .35
20	63	0.3548	0.3613	− .25	56	0.2670	0.2730	− .23
22	32	0.7345	0.7372	− .10	86	0.8952	0.8875	+ .30
24	78	0.2908	0.3028	− .46	41	0.3322	0.3270	+ .20
25	47	0.3902	0.3932	− .12	72	0.2398	0.2425	− .10
34	55	0.1102	0.1050	+ .20	64	0.5210	0.5260	− .19

PLATE II. *y* MEASUREMENTS

Star	*y* direct				*y* reversed			
	Line of Scale	½ *m* or Scale *minus* Star		Mitchell *minus* Harpham	Line of Scale	½ *m* or Scale *minus* Star		Mitchell *minus* Harpham
		Mitchell	Harpham			Mitchell	Harpham	
1	59	0.0062	0.0120	$-0\overset{r}{.}22$	60	0.6192	0.6200	$-0\overset{r}{.}03$
5	60	0.9275	0.9268	+ .03	58	0.6995	0.7030	− .14
16	67	0.4052	0.4055	− .01	52	0.2218	0.2268	− .19
17	55	0.5232	0.5250	− .07	64	0.1052	0.1118	− .25
17	55	0.5162	0.5215	− .20	64	0.1025	0.1090	− .25
19	51	0.1590	0.1600	− .04	68	0.4688	0.4615	+ .28
20	58	0.5300	0.5392	− .36	61	0.0895	0.0970	− .29
22	27	0.9185	0.9162	+ .09	91	0.6895	0.6948	− .20
24	73	0.4708	0.4755	− .18	46	0.1520	0.1550	− .12
25	42	0.5688	0.5692	− .02	77	0.0548	0.0610	− .24
34	50	0.2678	0.2505	+ .67	69	0.3752	0.3702	+ .19

PLATE III. *y* MEASUREMENTS

Star	*y* direct				*y* reversed			
	Line of Scale	½ *m* or Scale *minus* Star		Mitchell *minus* Harpham	Line of Scale	½ *m* or Scale *minus* Star		Mitchell *minus* Harpham
		Mitchell	Harpham			Mitchell	Harpham	
1	58	0.7968	0.7942	$+0\overset{r}{.}10$	60	0.8435	0.8385	$+0\overset{r}{.}19$
5	60	0.7118	0.7138	− .08	58	0.9208	0.9152	+ .22
16	67	0.2162	0.2175	− .05	52	0.4125	0.4122	+ .01
17	55	0.3252	0.3215	+ .14	64	0.3112	0.3112	.00
17	55	0.3228	0.3270	− .16	64	0.3070	0.3082	− .05
19	50	0.9778	0.9730	+ .19	68	0.6572	0.6620	− .19
20	58	0.3450	0.3482	− .12	61	0.2865	0.2898	− .13
22	27	0.7512	0.7482	+ .12	91	0.8820	0.8810	+ .04
24	73	0.3015	0.2998	+ .07	46	0.3308	0.3302	+ .02
25	42	0.3985	0.4020	− .14	77	0.2378	0.2362	+ .06
34	50	0.1028	0.1015	+ .05	69	0.5342	0.5402	− .23

These measures were corrected for division errors of the millimeter scale and errors of the micrometer screw, using the correction tables employed by Dr. Schlesinger in the computation of the *Praesepe* measures. The run of the micrometer screw was practically negligible. The corrected coördinates are given in Table III, where the quantities are in millimeters.

TABLE III.

PLATE I. CORRECTED COÖRDINATES.

Star	x			y		
	Direct	Reversed	Mean	Direct	Reversed	Mean
1	−40.5066	.5126	−40.5096	+ 3.4236	.4334	+ 3.4285
5	−25.5074	.5066	−25.5070	+ 5.3643	.3692	+ 5.3668
16	− 1.2525	.2504	− 1.2514	+11.8832	.8883	+11.8858
17	0.0000	:0000	0.0000	0.0000	.0000	0.0000
19	+ 1.2938	.2994	+ 1.2966	− 4.3666	.3630	− 4.3648
20	+ 4.1660	.1742	+ 4.1701	+ 3.0202	.0232	+ 3.0217
22	+ 5.0686	.0705	+ 5.0696	−27.6060	.5956	−27.6008
24	+ 8.7476	.7506	+ 8.7491	+17.9574	.9647	+17.9610
25	+12.8103	.8125	+12.8114	−12.9513	.9496	−12.9504
34	+32.6615	.6572	+32.6594	− 5.2328	.2319	− 5.2324

PLATE II. CORRECTED COÖRDINATES.

Star	x			y		
	Direct	Reversed	Mean	Direct	Reversed	Mean
1	−40.4908	.4943	−40.4926	+ 3.4889	.4898	+ 3.4894
5	−25.4970	.4946	−25.4958	+ 5.4058	.4078	+ 5.4068
16	− 1.2398	.2426	− 1.2412	+11.8862	.8850	+11.8856
17	0.0000	.0000	0.0000	0.0000	0000	0.0000
19	+ 1.2919	.2921	+ 1.2920	− 4.3625	.3578	− 4.3602
20	+ 4.1637	.1616	+ 4.1626	+ 3.0141	.0150	+ 3.0146
22	+ 5.0421	.0416	+ 5.0418	−27.6042	.5820	−27.5931
24	+ 8.7696	.7626	+ 8.7661	+17.9536	.9566	+17.9551
25	+12.7856	.7874	+12.7865	−12.9528	.9503	−12.9516
34	+32.6180	.6229	+32.6204	− 5.2640	.2659	− 5.2650

PLATE III. CORRECTED COÖRDINATES.

Star	x			y		
	Direct	Reversed	Mean	Direct	Reversed	Mean
1	−40.4800	.4922	−40.4861	+ 3.4716	.4707	+ 3.4712
5	−25.4905	.4970	−25.4938	+ 5.3886	.3934	+ 5.3910
16	− 1.2500	.2474	− 1.2487	+11.8950	.8992	+11.8971
17	0.0000	.0000	0.0000	0.0000	0000	0.0000
19	+ 1.2926	.3020	+ 1.2973	− 4.3520	.3499	− 4.3510
20	+ 4.1552	.1629	+ 4.1590	+ 3.0234	.0224	+ 3.0229
22	+ 5.0613	.0680	+ 5.0646	−27.5747	.5691	−27.5719
24	+ 8.7542	.7567	+ 8.7554	+17.9784	.9818	+17.9801
25	+12.7918	.7946	+12.7932	−12.9244	.9272	−12.9258
34	+32.6131	.6224	+32.6178	− 5.2238	.2282	− 5.2260

The focal length of the Crossley reflector[1] is 17 feet, 6.1 inches, and the approximate scale value of the photographs is therefore 1 mm=38.″65.

Multiplying the corrected x and y coördinates in Table III by 38.″65 sec δ_0 and 38.″65 respectively (where δ_0 is the declination of the central star), we obtain the quantities X sec δ_0 and Y.

Coefficients for computing refraction are given in the following table:

TABLE IV.

REFRACTION COEFFICIENTS.

Plate	M_x	N_x	M_y	N_y
1	+ 0.000249	− 0.000005	+ 0.000013	+ 0.000261
2	+ 250	− 7	+ 21	+ 260
3	+ 367	+ 7	− 135	+ 287

Four constants are necessary for the reduction of each plate. These constants are the corrections for scale value, orientation, and the two errors of the plate center. Put:

p = the correction to the scale value, so that 1 mm at the center of the plate corresponds to 38.″65 $(1+p)$;

r = the orientation correction, or the sine of the angle through which the axes must be rotated, measured in the direction of decreasing position angles;

k and c = the number of seconds of arc through which the coördinate axes must be moved in the direction of decreasing right ascensions and declinations respectively.

We then have from each known star a pair of equations of the form:

$$pX+rY+k+n_x=0$$
$$pY-rX+c+n_y=0$$

In these equations n_x and n_y are computed as follows, putting:

$\Delta\alpha$, $\Delta\delta$ = excess of the right ascension and declination of any star over the corresponding coördinates of the central star.

Then

$n_x \sec\delta_0 = X \sec\delta_0$ *plus* corrections for transformation and refraction *minus* $\Delta\alpha$;

$n_y = Y$ *plus* corrections for transformation and refraction *minus* $\Delta\delta$.

The right ascensions and declinations for computing $\Delta\alpha$ and $\Delta\delta$ were taken from Jacoby's *second* reduction of the Rutherfurd *Pleiades* photographs, the results of which are now in course of publication. The adopted positions are given in Table V.

[1] KEELER, "The Crossley Reflector," ASTROPHYSICAL JOURNAL, **11**, 325, 1900.

TABLE V.

ADOPTED POSITIONS OF THE STARS.

Star	α 1873.0	δ 1873.0
1	54° 2′ 36″.12	23° 58′ 12″.60
5	13 9.72	59 32.06
16	30 14.64	24 3 49.04
17	31 10.02	23 56 9.67
19	32 5.97	53 21.38
20	34 5.16	58 7.09
22	34 49.91	38 23.69
24	37 17.05	24 7 45.65
25	40 13.85	23 47 50.60
34	54 12.10	52 51.96

It is not necessary to bring these positions up to the date of the plates, as precession, nutation, and aberration are taken into account by the constants, p and r. The right ascension and declination of the central star, α_0 and δ_0, used in determining the refraction were, however, brought up to 1900.0 by precession.

These ten known stars furnish the following equations, in which the coefficients of p and r have been divided by 100, for convenience in making the solution.

RIGHT ASCENSIONS. DECLINATIONS.

Plate I

Star	$+ \quad pX + rY + k + n_x = 0$	Residual	$+ \quad pY - rX + c + n_y = 0$	Residual
1	$-15.7p + 1.3r + k - 1''.21 = 0$	$-0''.74$	$+1.3p + 15.7r + c + 6.95 = 0$	$-0''.13$
5	$-9.9p + 2.1r + k + 0.10 = 0$	$-.15$	$+2.1p + 9.9r + c + 4.02 = 0$	$-.48$
16	$-0.5p + 4.6r + k + 2.16 = 0$	$+.19$	$+4.6p + 0.5r + c + 0.14 = 0$	$-.25$
17	$0p + .0r + k + 0 = 0$	$-.17$	$0p \quad 0r + c + 0 = 0$	$+.12$
19	$+0.5p - 1.7r + k - 1.00 = 0$	$+.09$	$-1.7p - 1.5r + c - 0.45 = 0$	$+.01$
20	$+1.6p + 1.2r + k + 1.30 = 0$	$+.76$	$+1.2p - 1.6r + c - 0.63 = 0$	$+.15$
22	$+2.0p - 10.7r + k - 5.29 = 0$	$-.47$	$-10.7p - 2.0r + c - 1.12 = 0$	$+.57$
24	$+3.4p + 6.9r + k + 3.53 = 0$	$+.28$	$+6.9p - 3.4r + c - 1.73 = 0$	$-.50$
25	$+5.0p - 5.0r + k - 1.91 = 0$	$+.14$	$-5.0p - 5.0r + c - 1.86 = 0$	$+.84$
34	$+12.6p - 2.0r + k - 0.18 = 0$	$+.04$	$-2.0p - 12.6r + c - 6.27 = 0$	$-.31$

Plate II

Star	$+ \quad pX + rY + k + n_x = 0$	Residual	$+ \quad pY - rX + c + n_y = 0$	Residual
1	$-15.7p + 1.3r + k - 0''.55 = 0$	$-0''.77$	$+1.3p + 15.7r + c + 9.30 = 0$	$+0''.24$
5	$-9.9p + 2.1r + k + 0.54 = 0$	$-.24$	$+2.1p + 9.9r + c + 5.56 = 0$	$-.22$
16	$-0.5p + 4.6r + k + 2.55 = 0$	$+.19$	$+4.6p + 0.5r + c + 0.13 = 0$	$-.33$
17	$0p + 0r + k + 0 = 0$	$+.25$	$0p \quad 0r + c + 0 = 0$	$-.10$
19	$+0.5p - 1.7r + k - 1.17 = 0$	$+.04$	$-1.7p - 0.5r + c - 0.27 = 0$	$-.06$
20	$+1.6p + 1.2r + k + 1.00 = 0$	$+.54$	$+1.2p - 1.6r + c - 0.91 = 0$	$-.12$
22	$+2.0p - 10.7r + k - 6.36 = 0$	$-.05$	$-10.7p - 2.0r + c - 0.82 = 0$	$+.35$
24	$+3.4p + 6.9r + k + 4.19 = 0$	$+.45$	$+6.9p - 3.4r + c - 1.95 = 0$	$-.23$
25	$+5.0p - 5.0r + k - 2.87 = 0$	$+.14$	$-5.0p - 5.0r + c - 1.90 = 0$	$+.88$
34	$+12.6p - 2.0r + k - 1.68 = 0$	$-.51$	$-2.0p - 12.6r + c - 7.55 = 0$	$-.45$

	RIGHT ASCENSIONS. Plate III		DECLINATIONS. Plate III	
1	$-15.6p+1.3r+k-0''.47=0$	$-0''.78$	$+1.3p+15.6r+c+8.86=0$	$+0''.36$
5	$-9.9p+2.1r+k+0.49=0$	$-.29$	$+2.1p+9.9r+c+5.13=0$	$-.47$
16	$-0.5p+4.6r+k+2.26=0$	$+.11$	$+4.6p+0.5r+c+0.59=0$	$-.25$
17	$0p+0r+k+0=0$	$+.19$	$0p\ 0r+c+0=0$	$-.53$
19	$+0.5p-1.7r+k-0.96=0$	$+.09$	$-1.7p-0.5r+c+0.06=0$	$-.19$
20	$+1.6p+1.2r+k+0.89=0$	$+.45$	$+1.2p-1.6r+c-0.60=0$	$-.32$
22	$+2.0p-10.7r+k-5.47=0$	$+.16$	$-10.7p-2.0r+c-0.06=0$	$+.54$
24	$+3.4p+6.9r+k+3.83=0$	$+.46$	$+6.9p-3.4r+c-1.02=0$	$+.12$
25	$+5.0p-5.0r+k-2.56=0$	$+.13$	$-5.0p-4.9r+c-0.99=0$	$+1.03$
34	$+12.6p-2.0r+k-1.64=0$	$-.56$	$-2.0p-12.6r+c-6.24=0$	$-.32$

Solving these equations by least squares the following results are obtained:

TABLE VI.

Plate	p	r	Probable error of p and r	k	c	Probable error of k and c	$[v\,v]$	Probable error of one equation of weight unity
1	-0.000619	-0.004537	±0.000110	$+0''.0941$	$+0''.1199$	$\pm0''.0963$	$3''.2647$	$\pm0''.3046$
2	-0.000175	-0.005692	±0.000103	$+0.2451$	-0.0965	±0.0911	2.9222	±0.2882
3	-0.000104	-0.005104	±0.000119	$+0.1935$	-0.5254	±0.1043	3.8207	±0.3299

Collecting the residuals for comparison we have the second, third, and fourth columns of Table VII for the right ascensions and corresponding columns for the declinations.

TABLE VII.

Star	Right ascensions: Plate I	II	III	Mean	Plate *minus* mean I	II	III	Declinations: Plate I	II	III	Mean	Plate *minus* mean I	II	III
1	−.74	−.77	−.78	−.76	+.02	−.01	−.02	−.13	+.24	+.36	+.16	−.29	+.08	+.20
5	−.15	−.24	−.29	−.23	+.08	−.01	−.06	−.48	−.22	−.47	−.39	−.09	+.17	−.08
16	+.19	+.19	+.11	+.16	+.03	+.03	−.05	−.25	−.33	−.25	−.28	+.03	−.05	+.03
17	+.09	+.25	+.19	+.18	−.09	+.07	+.01	+.12	−.10	−.53	−.17	+.29	+.07	−.36
19	−.17	+.04	+.09	−.01	−.16	+.05	+.10	+.01	−.06	−.19	−.08	+.09	+.02	−.11
20	+.76	+.54	+.45	+.58	+.18	−.04	−.13	+.15	−.12	−.32	−.10	+.25	−.02	−.22
22	−.47	−.05	+.16	−.12	−.35	+.07	+.28	+.57	+.35	+.54	+.49	+.08	−.14	+.05
24	+.28	+.45	+.46	+.40	−.12	+.05	+.06	−.50	−.23	+.12	−.20	−.30	−.03	+.32
25	+.14	+.14	+.13	+.14	.00	.00	−.01	+.84	+.88	+1.03	+.92	−.08	−.04	+.11
34	+.04	−.51	−.56	−.34	+.38	−.17	−.22	−.31	−.45	−.32	−.36	+.05	−.11	+.04

These residuals are to be regarded as errors of the Lick plates on the assumption that the Rutherfurd results are absolutely correct, and that there has been no proper motion between 1873, the date of the Rutherfurd photographs, and 1900, the date of the Lick plates. Now, whatever may be the errors of the adopted Rutherfurd positions, and whatever may have been the effects of proper motion, the residuals ought to come out the same from each of the three Lick plates. Therefore the "mean residual" from the three plates has been computed for each star, and the divergences of the residuals (individual plate *minus* mean) set down in the last three columns of the table.

These latter residual numbers now give us a measure of the precision with which the Lick plates reproduce their own errors. Computing the sums of the squares of these numbers we find for:

Plate 1 - - - -	0ʳ.7082
2 - - - -	0.1241
3 - - - -	0.5200

whereas the original residuals gave, according to Table VI:

Plate 1 - - - -	3ʳ.2647
2 - - - -	2.9222
3 - - - -	3.8207

We see, therefore, that the agreement *inter se* of the Lick plates is much better than their accord with the Rutherfurd results; and this is favorable to the Crossley reflector. For the desideratum in an instrument is the ability to reproduce its own errors every time it is used, rather than that these errors should be extremely small. Nor should we ascribe the differences, "Lick *minus* Rutherfurd," to errors of the Lick instrument alone; since we have already pointed out that they are due in part to proper motion, and errors of the Rutherfurd catalogue.

Extended calculations have been made to ascertain whether the agreement between the Lick and Rutherfurd photographs can be improved by correcting the latter with proper motions derived from comparisons of the Königsberg and Yale heliometer measures. But these calculations have resulted unsuccessfully. Similarly, new least square solutions omitting two stars having large residuals failed to improve the result.

A computation of the probable error of the Lick plate measures from the data of Table II was also made. The method used was the same as that employed by Dr. Schlesinger.[1] It was found that the probable error of a final coördinate was ±0ʳ.07 for X and ±0ʳ.04 for Y. Dr. Schlesinger's corresponding values for the Rutherfurd *Praesepe* plates were respectively ±0ʳ.030 and ±0ʳ.025.

[1] *Annals N.Y. Acad. Sci.*, 10:272, or *Contrib. Observ. Columbia Univ.*, No. 15, p. 272.

Our final conclusion from all the evidence is therefore as follows: The Crossley plates have star images inferior to the Rutherfurd plates; but the distortion of the field is certainly small throughout a radius of about half a degree. Moreover, if there is a small distortion, it is very nearly constant in the same part of the field of different plates, even when taken at widely different hour angles.

MEASUREMENT AND REDUCTION OF A PHOTOGRAPH OF EROS MADE WITH THE CROSSLEY REFLECTOR OF THE LICK OBSERVATORY.

This plate was made September 19, 1900, with six successive exposures, two of five seconds duration, two of ten seconds and two of thirty seconds. The measures and reductions, however, include only the first and third exposures, of five and ten seconds, respectively. A microscopic examination of the plate showed that the ten-second images of *Eros* are quite sufficiently well defined for measurement. Even the five-second images are measurable easily; while the thirty-second ones are somewhat over-exposed.

The following are the particulars relating to the two exposures measured:

Exposures		Middle of exposure	Duration
1st	Pacific standard time	$14^h\ 45^m\ 17.5^s$	5^s
3d	" " "	14 45 45.0	10

Barometer and thermometer readings were not furnished, but the refraction will not be influenced appreciably, as the plate was taken very near the zenith, and values could be assumed from those employed in the *Pleiades* reductions.

Ten stars and *Eros* were measured. The following tables contain the measures and the corrected coördinates in a form similar to that used for the *Pleiades* plates:

FIRST IMAGE. x MEASUREMENTS

Star	x direct				x reversed			
	Line of Scale	½ m or Scale *minus* Star		Mitchell *minus* Harpham	Line of Scale	½ m or Scale *minus* Star		Mitchell *minus* Harpham
		Mitchell	Harpham			Mitchell	Harpham	
1	113	+0.3465	+0.3618	−0.″59	6	+0.2835	+0.2822	+0.″05
2	111	0.2798	0.2840	−0.16	8	0.3615	0.3570	+0.17
3	97	0.4208	0.4285	−0.30	22	0.2185	0.2110	+0.29
4	96	0.3025	0.3050	−0.10	23	0.3398	0.3315	+0.32
5	69	0.0705	0.0650	+0.21	50	0.5632	0.5700	−0.26
5	69	0.0692	0.0590	+0.39	50	0.5700	0.5700	0.00
6	66	0.4415	0.4418	−0.01	53	0.1968	0.1985	−0.07
7	52	0.0838	0.0568	+1.04	67	0.5540	0.5628	−0.34
8	41	0.8710	0.8548	+0.63	77	0.7728	0.7690	+0.15
9	39	0.6745	0.6592	+0.59	80	−0.0270	−0.0182	−0.34
10	33	0.3765	0.3792	−0.10	86	+0.2638	+0.2595	+0.17
Eros	72	0.6775	0.6708	+0.26	47	−0.0280	−0.0280	00

FIRST IMAGE. *y* MEASUREMENTS

Star	*y* direct				*y* reversed			
	Line of Scale	½ m or Scale *minus* Star		Mitchell *minus* Harpham	Line of Scale	½ m or Scale *minus* Star		Mitchell *minus* Harpham
		Mitchell	Harpham			Mitchell	Harpham	
1	35	−0.0155	−0.0200	+0″.17	84	+0.6540	+0.6545	−0″.02
2	78	+0.0100	+0.0175	− .29	41	0.6258	0.6202	+ .22
3	46	0.8300	0.8352	− .20	72	0.8102	0.8018	+ .32
4	76	0.8170	0.8315	− .56	42	0.8082	0.8005	+ .30
5	58	0.7908	0.7968	− .23	60	0.8478	0.8447	+ .12
5	58	0.8192	0.7955	+ .91	60	0.8498	0.8408	+ .35
6	31	−0.0088	−0.0085	− .01	88	0.6478	0.6485	− .03
7	31	+0.4295	+0.4222	+ .28	88	0.2412	0.2165	+ .95
8	57	−0.0450	−0.0455	+ .02	62	0.6768	0.6745	+ .09
9	67	+0.2288	+0.2350	− .24	52	0.3928	0.3902	+ .10
10	61	0.1215	0.1045	+ .66	58	0.5158	0.5168	− .04
Eros	67	0.6232	0.6178	+ .21	51	0.0100	0.0118	− .07

THIRD IMAGE. *x* MEASUREMENTS

Star	*x* direct				*x* reversed			
	Line of Scale	½ m or Scale *minus* Star		Mitchell *minus* Harpham	Line of Scale	½ m or Scale *minus* Star		Mitchell *minus* Harpham
		Mitchell	Harpham			Mitchell	Harpham	
1	113	+0.2832	+0.2845	−0″.05	6	+0.3480	+0.3572	−0″.36
2	111	0.1950	0.2050	− .39	8	0.4302	0.4295	+ .03
3	97	0.3418	0.3632	− .83	22	0.2922	0.2852	+ .27
4	96	0.2198	0.2362	− .63	23	0.4108	0.4118	− .04
5	69	−0.0075	−0.0080	+ .02	50	0.6508	0.6490	+ .07
5	69	−0.0055	−0.0125	+ .27	50	0.6468	0.6495	− .10
6	66	+0.3645	+0.3752	− .41	53	9.2735	0.2735	.00
7	52	0.0180	0.0070	+ .42	67	0.6620	0.6675	− .21
8	41	0.7985	0.7750	+ .91	77	0.8410	0.8610	− .77
9	39	0.6070	0.6240	− .66	80	0.0532	0.0715	− .71
10	33	0.3310	0.3080	+ .89	86	0.3470	0.3400	+ .27
Eros	72	0.5928	0.5900	+ .11	47	0.0498	0.0540	− .16

THIRD IMAGE. *y* MEASUREMENTS.

Star	*y* direct				*y* reversed			
	Line of scale	⅛ or Scale *minus* Star		Mitchell *minus* Harpham	Line of scale	½ m or Scale *minus* Star		Mitchell *minus* Harpham
		Mitchell	Harpham			Mitchell	Harpham	
1	35	+0.4665	+0.4592	+0″.28	84	+0.1778	+0.1730	+0″.19
2	78	0.4940	0.4978	− .15	41	0.1430	0.1405	+ .10
3	47	0.3092	0.3090	+ .01	72	0.3330	0.3232	+ .38
4	77	9.3010	0.3172	− .63	42	0.3285	0.3225	+ .23
5	59	0.2602	0.2675	− .28	60	0.3690	0.3692	− .01
5	59	0.26 0	0.2690	− .04	60	0.3692	0.3710	− .07
6	31	0.4640	0.4705	− .25	88	0.1658	0.1728	− .27
7	31	0.9108	0.8895	+ .82	87	0.7958	0.7720	+ .92
8	57	0.4328	0.4318	+ .04	62	0.2052	0.2050	+ .01
9	67	0.7008	0.7105	− .37	51	0.9165	0.9198	− .13
10	61	0.6000	0.5878	+ .47	58	0.0495	0.0442	+ .20
Eros	68	0.1125	0.1125	.00	51	0.5225	0.5265	− .15

CORRECTED COÖRDINATES. FIRST IMAGE.

Star	*x*			*y*		
	Direct	Reversed	Mean	Direct	Reversed	Mean
1	−44.2809	.2859	−44.2833	−23 8179	.8086	−23.8133
2	−42.2102	.2104	−42.2103	+19.2144	.2239	+19.2192
3	−28.3534	.3521	−28.3528	−11.9698	.9616	−11.9657
4	−27.2337	.2316	−27.2327	+18.0250	.0422	+18.0336
5	0.0000	.0000	0.0000	0.0000	.0000	0.0000
6	+ 2.6245	.6314	+ 2.6280	−27.8090	.8017	−27.8054
7	+16.9983	.9944	+16.9964	−27.3751	.3830	−27.3790
8	+27.2077	.2061	+27.2069	− 1.8460	.8310	− 1.8385
9	+29.4030	.4124	+29.4077	+ 8.4332	.4541	+ 8.4436
10	+35.6917	.6944	+35.6931	+ 2.3133	.3287	+ 2.3210
Eros	− 3.6064	.5957	− 3.6010	+ 8.8213	.8342	+ 8.8278

CORRECTED COÖRDINATES. THIRD IMAGE.

Star	*x*			*y*		
	Direct	Reversed	Mean	Direct	Reversed	Mean
1	−44.2848	.2967	−44.2908	−23.8038	.8061	−23.8050
2	−42.2026	.2104	−42.2065	+19.2301	.2290	+19.2296
3	−28.3555	.3580	−28.3573	−11.9600	.9599	−11.9600
4	−27.2323	.2367	−27.2345	+18.0439	.0449	+18.0444
5	0.0000	.0000	0.0000	0.0000	.0000	0.0000
6	+ 2.6170	.6266	+ 2.6243	−27.7996	.7991	−27.7994
7	+16.9818	17.0200	+17.0009	−27.3673	.4130	−27.3902
8	+27.2096	.2055	+27.2076	− 1.8342	.8367	− 1.8354
9	+29.3801	.4166	+29.3984	+ 8.4404	.4537	+ 8.4471
10	+35.6758	.6955	+35.6857	+ 2.3276	.3220	+ 2.3248
Eros	− 3.5980	.5965	− 3.5972	+ 8.8483	.8469	+ 8.8476

The refraction coefficients were found to be as follows:

M_x	N_x	M_y	N_y
+0.000244	−0.000009	+0.000004	+0.000244

The positions of the ten stars were taken from the *A. G. C. Catalogue*, and are:

No.	A. G. C. No.	α 1875.0			δ 1875.0		
1	2318	2h	35m	24.s56	40°	7′	23.″4
2	2323		35	30.24		35	10.1
3	2333		36	18.27		15	8.7
4	2334		36	21.30		34	29.4
5	2351		37	53.75		22	55.3
6	2355		38	3.06		5	2.1
7	2366		38	51.40		5	17.5
8	2379		39	25.88		21	46.5
9	2380		39	33.42		28	24.2
10	2386		39	54.60		24	27.5

The following equations were formed from the above star-places and plate measures, using No. 3 as the central star:[1]

	First Image. Right Ascensions		First Image. Declinations	
Star	$+\ pX + rY + k + n_x = 0$	Residual	$+\ pY - rX + c + n_y = 0$	Residual
1	$-17.1p - 9.2r + k - 0.''75 = 0$	$+1.''67$	$-9.2p + 17.1r + c + 5.''31 = 0$	$+0.''91$
2	$-16.3p + 7.4r + k + 2.96 = 0$	$+0.42$	$+7.4p + 16.3r + c + 2.64 = 0$	-0.42
3	$-11.0p - 4.6r + k - 3.05 = 0$	-1.62	$-4.6p + 11.0r + c + 1.56 = 0$	-0.69
4	$-10.5p + 7.0r + k + 0.50 = 0$	-1.53	$+7.0p + 10.5r + c + 0.75 = 0$	-0.59
5	$0p \quad 0r + k \quad .0 = 0$	$+0.77$	$0p \quad 0r + c \quad 0 = 0$	$+1.37$
6	$+1.0p - 10.7r + k - 5.25 = 0$	-1.19	$-10.7p - 1.0r + c - 1.75 = 0$	-0.79
7	$+6.6p - 10.6r + k - 4.49 = 0$	-0.08	$-10.6p - 6.6r + c - 1.54 = 0$	$+1.12$
8	$+10.5p - 0.7r + k - 1.17 = 0$	$+0.50$	$-0.7p - 10.5r + c - 4.55 = 0$	-0.06
9	$+11.4p + 3.3r + k - 0.40 = 0$	$+0.12$	$+3.3p - 11.4r + c - 5.14 = 0$	-0.11
10	$+13.8p + 0.9r + k - 0.43 = 0$	$+0.98$	$+0.9p - 13.8r + c - 6.39 = 0$	-0.79

[1] It should be noted that a "central star" is not really necessary; in fact, theory requires that the measures be referred to the center of the plate, rather than to a star near that point. No appreciable error has, however, been introduced in the present work by the slight departure from theory involved in the use of a "central star."

	Third Image. Right Ascensions		Third Image. Declinations	
1	$-17.1p-9.2r+k-1''.05=0$	$+1''.58$	$-9.2p+17.1r+c+5''.63=0$	$+0''.96$
2	$-16.3p+7.4r+k+3.10=0$	$+0.62$	$+7.4p+16.3r+c+3.04=0$	-0.33
3	$-11.0p-4.6r+k-3.23=0$	-1.64	$-4.6p+11.0r+c+1.78=0$	-0.70
4	$-10.5p+7.0r+k+0.43=0$	-1.56	$+7.0p+10.5r+c+1.17=0$	-0.43
5	$0p\quad 0r+k\quad 0=0$	$+0.85$	$0p\quad 0r+c\quad 0=0$	$+1.23$
6	$+1.0p-10.7r+k-5.39=0$	-1.15	$-10.7p-1.0r+c-1.52=0$	-0.66
7	$+6.6p-10.6r+k-4.32=0$	$+0.25$	$-10.6p-6.6r+c-1.97=0$	$+0.64$
8	$+10.5p-0.7r+k-1.14=0$	$+0.59$	$-0.7p-10.5r+c-4.43=0$	$+0.03$
9	$+11.4p+3.3r+k-0.76=0$	-0.21	$+3.3p-11.4r+c-5.01=0$	-0.02
10	$+13.8p+0.9r+k-0.72=0$	$+0.72$	$+0.9p-13.8r+c-6.25=0$	-0.66

A solution of these equations gave the following results for the unknowns:

Image	p	r	Probable error of p and r	k	c	Probable error of k and c	$[vv]$	Probable error of 1 equation of weight unity
First..	+0.000660	−0.003020	±0.000172	$+0''.7651$	$+1''.3748$	$\pm 0''.2224$	17.3947	$\pm 1''.0427$
Third.	+0.000631	−0.003113	±0.000163	+0.8458	+1.2257	±0.2101	15.5225	±0.7033

The large probable errors, as compared with the *Pleiades* plates, are due in part to the smaller precision of the adopted star places. The *A. G. C. Catalogue* must, of course, be expected to furnish much less accurate relative positions than are obtained from a purely differential micrometric catalogue, such as that derived from the Rutherfurd *Pleiades* photographs. But some of the increase in the residuals is due to the star-images on the *Eros* plate. These are not so good as the Lick *Pleiades* images,[1] the probable errors of a final coördinate x and y being $\pm 0''.11$ and $\pm 0''.07$, whereas we found for the *Pleiades* only $\pm 0''.07$ and $\pm 0''.04$.

With the values of p, r, k, and c obtained above, the place of *Eros* was computed from the measured coördinates.[2]

The results are, for the equinox of 1875.0:

Place of *Eros*, 1875.0						
Image	α			δ		
First	39°	25′	$22''.74$	40°	28′	$37''.71$
Third			23.02			38.32
Mean ,....	39	25	22.88	40	28	38.02

[1] *Eros* plates taken October 6–12 and recently received, appear to show images considerably better than the plates discussed in this paper.

[2] See Dr. Schlesinger's paper already quoted. *Annals N. Y. Acad. Sci.*, **10**, 246, or *Contrib. Obsy. Col. Univ.*, No. 15.

This position of *Eros* was brought up to 1900.0 by precession, and corrected with the usual "reduction to apparent place." A further correction for parallax was then applied, using for the planet's distance a value interpolated from the Ephemeris of *Eros* published in the Berlin *Jahrbuch* for 1902. Parallax corrections were also computed for Barnard's observations, made with the 40-inch refractor of the Yerkes Observatory[1] on September 19, which was also the date of the Lick plate. These observations were further corrected by the addition of $+0^s.07$ and $+1''.30$ to Barnard's right ascension and declination, which should reduce the place of Barnard's comparison star to the mean system of the ten stars upon which the Lick position is based. The numerical values of these corrections are simply k sec. δ and c from the least squares computation of the Lick plate.

The observed positions, thus corrected for parallax, were compared with places interpolated from the Berlin *Jahrbuch* Ephemeris of *Eros*, taking account of the aberration time. The Berlin *Jahrbuch* Ephemeris was also compared with the Ephemeris published by the Paris *Eros* committee in their Circular No. 3, dated August 17, 1900. This Paris Ephemeris is corrected with the results of observations made at Paris August 4 and 7, 1900. The following are, then, the corrections required by the Berlin *Jahrbuch* Ephemeris for the date September 19.

Authority	α	δ
Lick Plate	$+13^s.24$	$+31''.8$
Barnard	$+13.23$	$+32.1$
Paris Ephemeris	$+13.64$	$+30.4$

The accord between the Lick plate and Barnard's observation is satisfactory, and the Paris Ephemeris also represents the observations very well.

[1] *Astronomical Journal*, No. 484

ON THE PRODUCTION OF A LINE SPECTRUM BY ANOMALOUS DISPERSION, AND ITS APPLICATION TO THE "FLASH SPECTRUM."

By R. W. Wood.

In a communication published in the *Proceedings of the Royal Academy of Sciences*, Amsterdam,[1] W. H. Julius makes the very brilliant suggestion that the "flash spectrum" seen immediately at totality may be due to photosphere light abnormally refracted in the atmosphere of metallic vapors surrounding the Sun: in other words, the light of the flash spectrum does not come from the reversing layer at all, but from the photosphere. The author shows that the light which will be thus abnormally refracted will be of wave-lengths almost identical with the wave-lengths which the metallic vapors are themselves capable of radiating. This beautiful theory not only explains the apparent shallowness of the reversing layer, a thing that has always puzzled astrophysicists, but it accounts for the extraordinary brilliancy of the lines.

I have succeeded in producing such a flash spectrum by an arrangement in which I have endeavored to imitate as closely as possible the conditions supposed to exist at the surface of the Sun: in brief, I have obtained a spectrum of bright lines, with light from a source showing a continuous spectrum, by means of anomalous dispersion in an incandescent metallic vapor.

The theory of Julius supposes the Sun to be surrounded by an atmosphere of metallic vapors, the density and refractive index of which decrease with increasing distance from the surface. In this atmosphere the rays of light coming from the photosphere will move in curved paths similar to those of rays in our own atmosphere.

The refractive index is, however, very small except for wave-lengths very near those which are absorbed by the vapor, consequently the light most strongly refracted, if it could be sorted

[1] See also Astrophysical Journal, **12**, 185.

out and examined with the spectroscope, would resemble very closely the light emitted by the vapors. Julius shows that this sorting out of the more refrangible rays may account for the bright line spectrum usually attributed to the reversing layer, these rays moving in curved paths in the solar atmosphere, thus reaching us after the photosphere has been hidden by the Moon.

For the reproduction of the phenomenon in the laboratory it is necessary to form an atmosphere of metallic vapor in which the refractive index changes rapidly from layer to layer. This I succeeded in accomplishing by allowing the flame of a Bunsen burner fed with metallic sodium to play against the under side of a white plaster plate. On looking along the surface of the plate it was seen that a dark space existed between the flame and the cold surface, resembling somewhat the dark space surrounding the cathode of a Crookes' tube. It seemed highly probable that, inasmuch as the temperature of the flame was lowered to such a degree by contact with the plate, the density of the sodium vapor would increase very rapidly from the surface of the plate downward. The change may of course be abrupt instead of progressive, though I am inclined to favor the latter supposition. In either case the action will be practically the same, the case being similar to the transition from a curved ray to a broken line ray, as the change of the index of the medium becomes less gradual. The under surface of the plaster plate being covered with a non-homogeneous layer of sodium vapor, a spot at the edge of the flame was illuminated with sunlight concentrated by a large mirror. This spot radiated white light in every direction and corresponded to the incandescent photosphere of the Sun (Fig. 1). A telescope provided with an objective direct vision prism was directed toward the white spot and moved into such a position that, owing to the reduction in the width of the source of light by foreshortening, the Fraunhofer lines appeared in the spectrum. This represented the stage of an eclipse when only the thin crescent of the Sun is visible. The sodium flame appeared superposed on the spectrum, of course.

On moving the spectroscope until it was well inside of the plane of the illuminated surface and feeding the flame with fresh sodium, the solar spectrum vanished and there suddenly blazed out two narrow bright yellow lines, almost exactly in the place of the sodium lines, as is shown in Fig. 2, in which the inverted

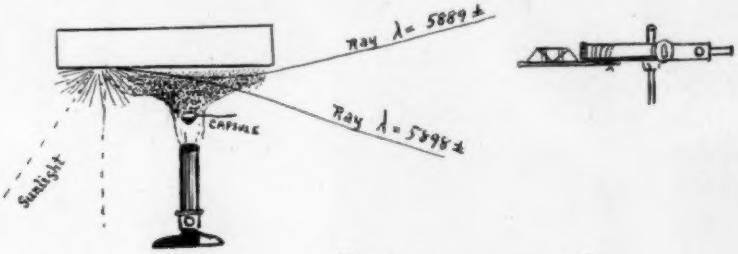


FIG. 1.

sodium flame appears on the continuous spectrum. Cutting off the sunlight with a screen caused the instant disappearance of the lines.

Repeating the experiment I found that the bright lines came into view on the sides of the sodium lines toward the blue, that is to say, it is light for which the medium has an abnormally low refractive index that is bent around the edge of the plate and enters the instrument. This is precisely what we should expect, for sodium vapor has a refractive

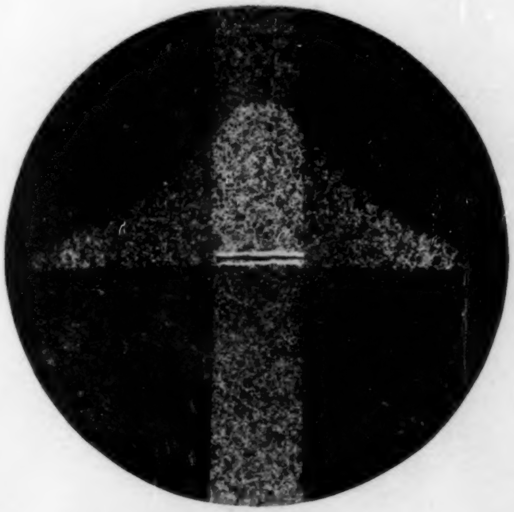


FIG. 2.
Flash Spectrum of Sodium produced by Anomalous Dispersion.

index of less than 1 for waves slightly shorter than D_1 and D_2, as was shown by Julius in his paper. The rays then will be concave upward in a medium in which the refractive index varies

as I have supposed it to vary in the present case. If the sodium vapor is very dense we see only a single bright line bordering D_2, owing to the complete absorption of the light between the lines.

I next instituted a search for the light of a wave-length slightly greater than that of the sodium lines. For these waves the vapor has a refractive index greater than 1, consequently the rays will be concave downward in the layer of vapor. If we move our prismatic telescope down in a search for these rays the solar spectrum will appear and drown out everything, but if we set up a screen (shown in Fig. 3) in such a position as to

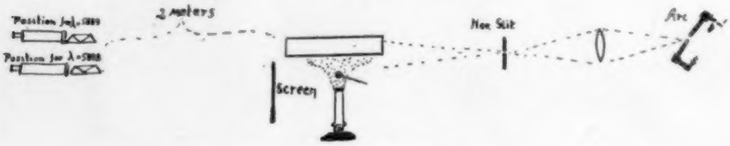


FIG. 3.

just cut off the light from the illuminated spot, and feed the flame with sodium, we shall presently see bright lines appear on the side of the sodium lines towards the red. In this case when the vapor is dense we get only a single line bordering D_1. The paths of these rays is indicated (on an exaggerated scale) in Fig. 1. The arrangement described is inconvenient in many ways to work with, and I accordingly modified it in the following way.

The light of an arc lamp is focused on a horizontal slit, and a flat metal plate supported so that the plane in which its under surface lies coincides with the plane of the slit. The plate should be an inch or so thick, with a fairly level surface. At a distance of about two meters a telescope provided with a prism (direct-vision if possible), arranged so as to give a vertical spectrum, is placed at such a height that the prism barely catches the rays coming from the slit and grazing the surface of the plate, Fig. 3. On looking into the telescope we see a bright continuous spectrum, and the telescope is to be raised until this becomes quite faint. The Bunsen burner beneath the plate is now to be lighted and a

bit of sodium, in a small iron capsule, introduced into the center of the flame. The results obtained are practically identical with those which have been described. The flash spectrum of potassium has been obtained in a similar manner, consisting of lines in the extreme red from one to three in number according to the density of the vapor and position of the telescope. Fair results have also been obtained with thallium.

I am now arranging apparatus by which I hope to obtain similar flash spectra by the dispersion of vapors exhibiting more complicated absorption spectra than sodium. If these experiments are successful much can be learned by comparing the flash spectra with the emission spectra. If it be found that certain lines are absent in the flash which are present in the emission spectra, interesting comparisons can be made with photographs of the actual flash-spectrum of the Sun. Julius applies the anomalous dispersion theory to the prominences as well as to the reversing layer. I have succeeded in producing such a phenomenon with sodium vapor, a wavering flame of intense brilliancy perfectly sharp when seen through a prism of high dispersion, and yet shining wholly by refracted light coming from the crater of an arc light. I am also engaged in making accurate determinations of the dispersion of metallic vapors by means of a metal prism of 45 degrees furnished with mica windows. The prism is filled with hydrogen and the metal—say sodium—vaporized in this atmosphere by the application of heat. The results obtained in this way are far superior to those yielded by prismatic flames. The angle of the prism is accurately known, and it is filled with non-luminous sodium vapor of uniform density and under known conditions of temperature and pressure. With this I have obtained much greater curvature of the spectrum in the vicinity of the absorption lines than that figured by Julius, and the spectrum is perfectly steady, instead of fluttering, as is the case when the deviation is effected by means of a sodium flame of prismatic form. The work along these lines will be reported in a subsequent paper.

UNIVERSITY OF WISCONSIN,
December 22, 1900.

THE NATURE OF THE SOLAR CORONA.

By R. W. Wood.

I propose in the following paper to discuss certain theories of the solar corona, and present the results of some recent experiments which I feel may have some bearing on the subject.

The most generally accepted theory of the corona attributes its continuous spectrum to light emitted in virtue of high temperature due to solar radiation,[1] with which is mixed a small amount of reflected sunlight producing traces of radial polarization. The recent work of Abbot, of the Smithsonian Institution, at Wadesboro, showing a cold corona, has set people to thinking about the old electrical theory, according to which the corona is regarded as a phenomenon akin to the aurora, and the light in Geissler tubes.

Certain of the experiments to be described later on indicate, it seems to me, that the absence of radiant heat offers no difficulty to the solid particle theory, being in fact precisely what we should expect. On the other hand, the presence of polarized light in the spectrum, and there is a good deal of it, judging from the strength of the Savart bands which I observed at the eclipse of last May, must be taken almost as proof positive that minute solid particles are present. Under no conditions has an electrically excited gas been found to emit polarized light, at least not to my knowledge. One or two cases in which the phenomenon was supposed to have been found, were shown to be spurious, the polarization resulting from reflection from the inner walls of the tube.

Going back now to the other theory: If the large amount of polarized light is reflected sunlight, why are not the Fraunhofer lines seen in the spectrum? I have called attention in a

[1] See paper by Scheiner and reply by Sir William Huggins in recent numbers of the Astrophysical Journal.

previous paper[1] to the fact that this may be because the oblique prism faces refuse transmission to the polarized light, which is the light in which the wave-lengths corresponding to the dark lines are absent. The certain knowledge that the lines are present or absent will be of great aid in formulating a satisfactory theory of the coronal light, and I trust that the long Sumatra eclipse will yield evidence in this direction. In the paper alluded to I suggested the use of a Nicol prism before the slit in such a position as to transmit the polarized radiations, the slit being set tangential to the Sun's edge, in which position the light entering the instrument is polarized in such a plane as to be transmitted by the prism faces. The light showing the dark lines would then be transmitted with undiminished intensity, while the emitted or non-polarized light would be reduced in intensity by one half. The great change in the ratio might easily be sufficient to bring out the dark lines. I feel sure that the experiment is worth trying at some future eclipse, for I have carried it out successfully in the laboratory with an artificial corona. It was found that a gas flame in a strong beam of sunlight shone with a pure bluish-white light, due to the reflection or rather scattering of the sunlight by the minute carbon particles.[2] A photograph of the flame with a spot illuminated by powerful convergent beams of sunlight is reproduced. It furnishes a beautiful proof of the existence of solid particles in the flame (Fig. 1.) The flame thus illuminated showed the Fraunhofer lines distinctly, but by reducing the intensity of the sunlight a point was reached at which they disappeared, and the spectrum appeared continuous. The light scattered by the flame was found to be *completely* plane-polarized in certain directions, giving us just the required conditions, namely particles emitting a continuous spectrum, and scattering a polarized solar spectrum. In front of the slit of the spectroscope a Nicol was arranged in

[1] "The Problem of the Daylight Observation of the Corona," ASTROPHYSICAL JOURNAL, November 1900.

[2] The reflection of light by flames I have since found has been observed before by Mr. Burch and also by Sir George Stokes.

such a manner that it could be drawn into and out of position by a cord. The Fraunhofer lines could be made to appear by sliding the Nicol in front of the slit, and disappear by drawing it away. While it does not by any means follow that the use of a Nicol on the actual corona will bring out the lines, the experiment seems to be well worth trying, as it would furnish further information regarding the relative intensity of the emitted and reflected light. Another interesting point is that the minute particles in the flame do not scatter the longer waves, the flame reflecting practically no red or orange light. Thus the Fraunhofer lines can only be traced up to about the D lines. By gradually reducing the intensity of the sunlight they disappear first in the yellow, then in the green, blue, and violet in succession. This indicates that our chances of detecting the lines in the spectrum of the corona will be greatest in the actinic part of the spectrum.

FIG. 1.

The preponderance of the shorter wave-lengths in the light scattered by the carbon particles in the flame can be shown in the following way. The light from the crater of an arc lamp is focused on a candle flame by means of a lens. We thus get a very small spot in the center of the flame illuminated with a powerful light rich in waves of all lengths. This candle flame is then photographed with an objective prism, on a plate made sensitive to the entire spectrum. We find that the spectrum of the flame has a bright line running through its center — the spectrum of the illuminated spot. This line, however, can only be traced as far as the yellow; there is absolutely no trace of it in the red and scarcely a trace in the orange. In the green it is many times brighter than the background, while in the violet it stands out strong on a black background, showing that the flame gives out very little violet light except in the spot where the image of the arc falls.

In making this negative a color filter was used during a part of the exposure to cut down the action of the blue and violet, while a record of the red end of the spectrum was being secured. The apparent absorption band between the yellow and green is due to the fact the plates are not absolutely orthochromatic, and has no bearing on the subject. Two of these photographs are reproduced in Fig. 2, a short and a long exposure.

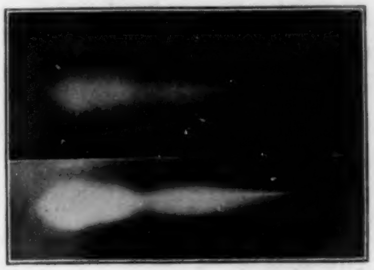


FIG. 2.

This inability of the particles in the flame to scatter or diffract the longer waves may explain the absence of heat radiations in the corona's spectrum, for we have only to assume that the particles are small in comparison to the wave-length. A determination of the distribution of energy in the corona spectrum would be useful in this connection. It would seem as if the red ought to be relatively feebler than in the solar spectrum.

This explanation of the absence of radiant heat does not apply to the emission spectrum due to the incandescence of the particles, but it seems to me to be very probable that, if the incandescence is due to solar radiation alone, the scattered or reflected light will be greatly in excess of the emitted light. The temperature of a body near the Sun has been recently calculated by Scheiner[1] and found to be about 4000 degrees at a distance of one half of the solar radius from the Sun's surface. Doubtless a body at this temperature even if very minute will emit a powerful light, but the fact appears to have been overlooked that a body placed in a radiation intense enough to produce this high temperature will—if it be large—reflect, or if small diffract an amount of light which will be commensurate with the amount emitted. It occurred to me that any determinations of the ratio of emitted to scattered light of a body brought

[1] ASTROPHYSICAL JOURNAL, July 1900.

to incandescence by solar radiation would perhaps throw some light on the problem. The question could, it seems to me, be at once settled if we had a burning glass capable of giving a temperature of 4000 degrees at its focus, as this is the upper limit placed by Scheiner for a small black spherical particle at a distance of less than half a radius from the surface of the Sun. The corona can, however, be followed to a distance of from two to three diameters, consequently we are not obliged to limit ourselves to particles in such proximity to the radiating surface. Using the values for W and σ adopted by Scheiner, which will be more apt to give too large than too small values, I have calculated the temperatures and intensities of radiation at various distances from the Sun. These are given in the following table. The distances from the surface are given in the first column, the intensity of the radiation (the intensity at the Earth's distance being unity) in the second, and the corresponding temperature of a black spherical body in the third.

Distance	Intensity of Radiation	Absolute Temp.
½ Radius	23000	4160°
1 Radius	11600	3500
1 Diameter	5160	2870
2 Diameters	1860	2200
3 Diameters	923	1860

I had at my disposal a silver concave mirror of short focus, the area of which was 803 sq. cm, which gave an image of the Sun 9 mm in diameter, or of area 0.63 sq. cm. The ratio of these areas is the measure of the intensity of the radiation at the focus, and is 1270, corresponding to that at a distance of between two and three solar diameters. The sunlight, however, passed through the window and through the glass of the mirror, consequently a considerable amount of energy is lost by reflection and absorption. That due to the former can be calculated by Fresnel's formula, and is found to be, for four glass surfaces, at normal incidence about 0.2 of the whole. This will bring our intensity down to a little less than 1000, which, neglecting atmospheric absorption, corresponds to the intensity at a distance of three solar diameters.

The temperature at the focus was found to be 1100°, copper being melted with ease. The mirror was provided with a diaphragm, as shown in Fig. 3, by which the intensity of the radiation at the focus could be controlled. One side of the square aperture was graduated to centimeters, and the area of the exposed portion of the mirror could be at once determined. Determinations of the temperature at the focus, with different apertures, were made in the following way. A number of alloys and metals fusing at temperatures varying from 185° C. to 1100° C. were rolled into sheets of foil of uniform thickness, from which disks, a trifle smaller than the image of the Sun at the focus of the mirror, were cut. Each disk was supported by a very thin strip of the same metal, to lessen as much as possible losses by conduction. The disks were smoked in a candle flame and the number of square centimeters of mirror necessary to fuse each determined. The radiating surface was of course double the area of the surface receiving the radiation, while in the case of a spherical particle it is four times as large; consequently the temperatures taken by the disks were much higher than we should have in the case of solid particles. This was easily shown by a temperature calibration of the mirror made with a mercury thermometer. For example, 100 square centimeters of mirror surface were sufficient to melt the zinc disk (415°), while 200 sq. cm radiating against the thermometer bulb, produced a temperature of only 270°.

FIG. 3.

Though the temperature at the focus was not as high as would be desirable for a conclusive test, it seemed that some notion of the ratio between the emitted and diffracted light might be obtained by bringing bodies brought to full incandescence by other means into the focus. I have already mentioned the experiment of illuminating a flame with the concentrated beam. Here we have small black particles at a temperature of

say 2000° (based on the fact that fine platinum wire fuses in the flame). It was found that in the case of a candle flame the radiation from 25 sq. cm of the mirror's surface caused the Fraunhofer lines to appear in the spectrum of the flame. This means that solar radiation only forty times as intense as the radiation at the Earth's distance will be scattered to such an extent by small particles, themselves radiating in virtue of a temperature of at least 1200, and possibly 2000 degrees, that the diffracted light is at strong as the emitted. This last statement is based on Hastings' statement in his eclipse report that the Fraunhofer lines remain visible until the sunlight is diluted with rather more than an equal amount of continuous spectrum light.

In the case of a flame of illuminating gas a radiation of intensity 230 was necessary to bring out the lines in the spectrum. This seemed somewhat surprising at first sight, for the temperature cannot be very different in the two cases, and the color of the flame was about the same as that of the candle flame. I found by a little experimenting that the probable cause was to be found in the smaller size of the particles than in their higher temperature. A gas flame from a large aperture, say 3 mm in diameter would show the lines with an illumination from 121 sq. cm of mirror, while a flame from an aperture of say 0.5 mm, which could not be distinguished in appearance from the other flame, would not show the lines even when illuminated with the full radiation of the entire mirror (over 800 sq. cm). I can only explain this anomaly on the supposition that in one condition the carbon particles are set free in a much more finely divided state than in the other.

In Fig. 4 are reproduced photographs of a gas and candle flame close together, both traversed by the concentrated beam from the entire mirror. The gas flame is burning from a small aperture, and shows no trace of the illuminating beam, while the candle flame is intensely illuminated. The two prints are given of the negative to enable a better idea of the relative intensity of the candle flame and the spot of sunshine on it to be formed.

Summing up the results thus far it seems safe to assert that a radiation 40 times as strong as the radiation at the Earth's distance is sufficient to show the Fraunhofer lines in the spectrum of matter incandescent at say 1500° (to take an intermediate value). Now at the distance from the Sun taken by Scheiner, where the temperature will be 4000°, we have a radiation 23,000 times as intense as the radiation at the Earth's distance. The scattered light will increase directly in proportion to the intensity of the radiation, that is it will be 575 times as intense as in the case of a flame illuminated by light 40 times as bright as ordinary sunlight at the Earth's surface. Whether the intensity of the *emitted* light will increase in like proportion is the question on which the appearance of the Fraunhofer lines depends. If we could raise small particles to a temperature of 4000° and then illuminate them with sunlight of sufficient intensity to bring out the lines we should have a direct answer to the question, but this I have been unable to do.

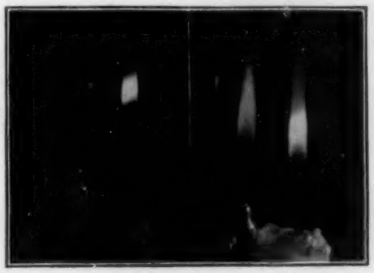


FIG. 4.

The temperature of the arc has been estimated at about 4000°, but the intensity of the radiation at the focus of my mirror is insufficient to accomplish the desired result in this case. I have experimented with various sources of light and then by comparing their intensities have been able to arrive at a fairly definite conclusion. The results of these experiments are given in the following table. The intensity of the solar radiation necessary to cause the appearance of the dark lines in the spectrum of the illuminated spot on the incandescent source, is expressed as before in terms of the normal solar radiation at the Earth's surface.

Source of Light	Intensity of Radiation
Welsbach mantle	8
Candle flame	40
Platinum at melting point	32
Calcium light	100
Carbon rod in oxy-hyd. flame	100
Illuminating gas flam	230

The Welsbach mantle is at a temperature of probably 1500° more or less: being a white substance it reflects strongly and a surprisingly feeble solar radiation is sufficient to impress the solar spectrum on it. Probably the carbon particles in the gas flame are at a temperature nearly as high, but it does not seem to me that we are justified in comparing the light diffused by a solid with the light diffracted by small particles. How much stress can be laid on the results obtained with sources of light in the form of compact solid masses I do not feel prepared to say. It is interesting to see that the same intensity is required for a white lime cylinder and a black carbon rod in the oxy-hydrogen flame. The lime cylinder is many times as bright as the carbon rod, but being white it reflects strongly. I have made photometric determinations which are recorded in my notes as follows. The arc light is 6 times as intense as a lime cylinder illuminated with the full aperture of the mirror. The arc is 36 times as bright as a carbon rod illuminated in the same way. It would appear from this that a radiation 36 times as intense as that produced by my mirror would be necessary to cause the arc to show the Fraunhofer lines, whereas a white substance at the same temperature would show them under an illumination 6 times that produced by the mirror. At the distance from the Sun's surface taken by Scheiner in his calculations the radiation would be some 23 times stronger than that of the mirror. A black substance would then reflect and emit about the same amount, while a white substance would reflect much more than it emitted. Whether this will hold for very small particles or not I do not know. It is almost impossible to draw any conclusions regarding a body at such a high temperature as 4000°, but if we retreat to a greater distance from the Sun, say to a distance of 2 diameters, where the upper limit that we can assign to the temperature is only a little over 2000°, it is easier to apply the experimental data. Here the intensity of the solar radiation will be only about double that at the focus of the mirror, and practically everything emitting light at this temperature (melting platinum, carbon and lime in the oxy-hydrogen flame) showed

the Fraunhofer lines when illuminated with very much less than the full aperture of the mirror. The exception is the gas flame from a small aperture, but I am inclined to regard this as a case of particles too small to scatter any appreciable amount of light, rather than a direct temperature effect.

I am well aware that the results and arguments set forth in this paper are open to criticism from every side. There is probably no field of research in which there are so many pitfalls as that of radiation and emission. I feel that much more satissactory conclusions could be reached with a mirror which would give an intensity at the focus of say 10 times that with which I have worked. In the present case too much dangerous exterpolation is required in drawing conclusions. Some of the results seem to me to be interesting and if they prove suggestive in any way to others better qualified than myself to discuss the problem, I shall feel well repaid for the work.

The absence of radiant heat in the spectrum of the corona indicates apparently that the amount of light emitted in virtue of incandescence must be small. It has also been supposed to indicate an absence of reflected sunlight, but this we see can be explained by the small size of the particles. My present notion is that the path of least logical resistance is to assume that in the corona we have very minute particles shining principally by diffracted sunlight, and moving towards or away from the Sun with sufficient velocity to preclude the appearance of the Fraunhofer lines in the spectrum by Doppler's principle applied to the line of sight component of velocity.

The breadth of the "1474" corona line in Professor Campbell's photograph indicates the strong probability of internal motion in the corona, and I see no reason why this motion cannot be postulated for the diffracting particles as well as for the incandescent coronium vapor.

Much light can be thrown on the nature of the corona by a more complete study of its polarization. The Sumatra eclipse, on account of its long duration, will furnish exceptional opportunities for work of this sort. For preliminary work with

apparatus arranged for the study of the polarization of the corona, I believe that an artificial corona that I have recently devised will be found most useful. It resembles the real corona in a most striking manner, and is polarized in the same way. I have published a brief account of it in *Science*.

A rectangular glass tank about a foot square on the front and five or six inches wide, and a six-candle-power incandescent lamp are all that are necessary. The dimensions of the tank are not of much importance, a small aquarium being admirably adapted for the purpose. The tank should be nearly filled with clean water, and a spoonful or two (the right amount determined by experiment) of an alcoholic solution of mastic should be added. The mastic is at once thrown down as an exceedingly fine precipitate, giving the water a milky appearance. The wires leading to the lamp should be passed through a short glass tube, and the lamp fastened at a right angle to the end of the tube with sealing wax, taking care to make a tight joint, to prevent the water from entering the tube. Five or six strips of tin foil are now fastened with shellac along the sides of the lamp, leaving a space of from ½ to 1 mm between them. The strips should be of about the same width as the clear spaces. They are to be mounted in two groups on opposite sides of the lamp, and the rays passing between them produce the polar streamers. The proper number, width, and distribution of the strips necessary to produce the most realistic effect can be easily determined by experiment. A circular disk of metal a trifle larger than the lamp should be fastened to the tip of the lamp with sealing wax, or any soft, water-resisting cement; this cuts off the direct light of the lamp and represents the dark disk of the Moon. The whole is to be immersed in the tank, with the lamp in a horizontal position, and the metal disk close against the front glass plate. It is a good plan to have a rheostat in circuit with the lamp to regulate the intensity of the illumination. On turning on the current and seating ourselves in front of the tank, we shall see a most beautiful corona, caused by the scattering of the light of the lamp by the small particles of mastic suspended in the water. If we

look at it through a Nicol prism we shall find that it is radially polarized, a dark area appearing on each side of the lamp, which turns as we turn the Nicol. The illumination is not uniform around the lamp, owing to unsymmetrical distribution of the candle power, and this heightens the effect. If the polar streamers are found to be too sharply defined or too wide, the defect can be easily remedied by altering the tin-foil strips. The eclipse is not yet perfect, however, the illumination of the sky background being too white and too brilliant in comparison. By adding a solution of some bluish-green aniline dye (I used malachite-green) the sky can be given its weird color, and the corona brought out much more distinctly. If the proper amount of the dye be added, the sky can be strongly colored without apparently changing the color of the corona in the slightest degree, a rather surprising circumstance, since both are produced by the same means. We should have now a most beautiful and perfect reproduction of the wonderful atmosphere around the Sun, a corona of pure golden white light, with pearly luster and exquisite texture, the misty streamers stretching out until lost on the bluish-green background of the sky. The rifts or darker areas due to the unequal illumination are present, as well as the polar streamers. The effect is heightened if the eyes are partially closed.

FIG. 5.

A photograph of one of these artificial eclipses is reproduced in Fig. 5. Much of the fine detail present in the negative is lost in the print, and still more will doubtless go in the process of reproduction. The coronal streamers extend out much farther than is indicated by the photograph. No especial pains were taken to get the polar rays just right.

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A PRELIMINARY DETERMINATION OF THE MOTION OF THE SOLAR SYSTEM.

By W. W. Campbell.

The first investigation undertaken with the Mills spectrograph, in May 1895, related to the determination of the radial velocities in the system of *Saturn*.[1] It confirmed, in all respects, the noted results announced by Professor Keeler a few weeks earlier. Determinations of stellar velocities were now undertaken, and results of considerable accuracy were at once obtained. The observed velocities of a bright solar type star could be depended upon to fall within a range of five or six kilometers. However, it soon became apparent that the instrument contained many defects. Some of these, with their remedies, have been described in my article on "The Mills Spectrograph," in this Journal for October 1898; but the large majority were purely local, and do not call for special comment. The greater part of the first year was devoted to isolating and eliminating these defects; and it was not until the summer of 1896 that results considered satisfactory for publication were secured. Added precautions taken, and improvements made in the instrument and methods, have shown corresponding and gratifying increase of accuracy from year to year.

Following the methods of observation already described in this Journal, two thousand spectrograms have been secured since the summer of 1896. These include: plates of the solar spectrum for determining the camera focus and scale values; plates of stellar spectra for determining the focus of the 36-inch objective at different temperatures; plates of comparison spectra, etc.; perhaps one hundred stellar spectrograms rejected for cause, without measurement; and in the neighborhood of fifteen hundred satisfactory spectrograms of about three hundred and

[1] Astrophysical Journal, August 1895, pp. 127–135.

twenty-five stars, situated between the North Pole and Declination -30°. At least three or four hundred of these photographs relate to spectroscopic binaries, for some of which, such as ζ *Geminorum*, nearly fifty plates were needed.

It is not practicable to publish the observed velocities at the present time, for two reasons:

(A) Many of the plates have been only partially measured, and reduced by approximate methods. Experience shows that these approximate results may be changed as much as $1\frac{1}{2}$ km by the final measures and reductions, though the average change is much less.

(B) The reductions have not been based upon the definitive wave-lengths of the solar and comparison lines. These are not yet available, but they are expected soon.

Repeated requests have been made that the observations already secured should be used to determine the motion of the solar system with reference to the system of observed stars; and it is the purpose of this article to communicate the preliminary results of such an investigation.

Omitting several Type I stars whose lines could not be accurately measured, and some thirty spectroscopic and visual[1] binaries for whose centers of gravity the velocities are not yet known, there remain 280 stars available for determining the relative motion of our system. Inasmuch as this number is constantly increasing with the progress of the observations, and in a few years will, I hope, be doubled and include stars distributed over the entire sky, it did not seem necessary to form an equation of condition for each star. The 280 stars were divided into 80 groups, by combining neighboring stars into one group; taking the mean of their individual velocities as the velocity of the group. The data for each of the 80 groups are contained in the first four columns of Table I.

Let v be the observed speed of a star with reference to the solar system; V the Sun's speed with reference to the system of 280 observed stars; and D the angular distance of a star from

[1] Such as *Sirius, Procyon*. etc.

the apex of the solar motion. Then each star, or each group of stars, furnishes an equation of condition having the form:

$$V \cos D - v = 0. \qquad (1)$$

Let α_0, δ_0 be the coördinates of the apex, and α, δ those of the star; then we have $\cos D$ defined by the well-known equation for the distance between two stars or points,

TABLE I.

No. of Stars	Mean R. A.	Mean Dec.	Mean Observed Velocity	No. of Stars	Mean R. A.	Mean Dec.	Mean Observed Velocity
	h m		k		h m		k
2	0 26.5	−14.°0	+17.0	3	12 20.4	+43.°0	− 4.7
6	0 40.0	+57.8	−10.3	3	12 35.7	−22.5	− 1.7
5	0 45.0	+28.2	−24.2	3	12 48.1	+ 4.9	−17.0
3	1 23.0	−10.1	+12.7	2	13 6.2	+23.2	− 8.2
4	1 35.5	+43.9	+ 0.2	2	14 9.2	− 7.6	+ 3.0
3	1 43.7	+10.6	+ 7.3	5	14 14.9	+18.1	− 3.9
2	1 47.4	−19.0	+ 2.5	2	14 29.5	−25.6	+11.8
2	2 2.6	+24.2	−19.0	7	14 58.0	+29.6	−13.1
9	2 38.5	−14.1	− 3.2	4	15 2.4	+46.0	−28.5
3	2 54.4	+ 0.1	+21.7	4	15 23.2	+ 4.8	− 7.0
5	2 55.4	+50.2	+13.6	4	15 27.4	+65.8	−14.2
4	3 3.8	+41.2	+ 4.5	4	15 41.4	−12.9	− 1.0
3	3 16.9	+13.6	+ 6.7	4	16 17.1	−20.8	−11.1
3	3 45.8	− 9.2	−10.7	4	16 32.8	− 8.5	− 7.0
3	4 3.2	−17.3	+34.8	3	16 38.0	+13.3	−26.0
5	4 16.6	+18.0	+35.5	3	16 49.5	+35.9	−29.0
2	4 29.7	+79.8	− 3.0	3	17 44.7	+53.6	−22.3
4	4 46.8	+44.7	+ 8.2	4	17 53.6	+29.5	− 9.9
3	5 15.7	+37.4	+30.7	3	17 53.8	+ 5.3	− 7.5
2	5 17.0	+ 7.1	+22.0	6	18 18.6	− 7.2	− 5.4
4	5 23.5	−20.9	+ 1.8	6	18 36.4	−23.5	−23.5
3	5 32.6	− 7.5	− 0.7	3	18 38.6	+19.0	−28.5
3	5 51.5	+57.7	+ 5.0	2	18 43.0	+41.2	−20.5
4	6 18.1	+25.0	+24.8	6	18 48.9	+69.6	+ 6.4
2	6 41.0	−15.6	+50.5	4	19 42.1	+ 6.2	−19.9
3	6 43.3	+16.7	+ 6.3	5	19 42.6	+23.6	−11.6
3	7 32.4	+27.0	+11.3	4	20 3.6	+55.4	−44.2
2	7 33.8	−25.1	+42.0	3	20 19.3	− 9.1	− 8.7
2	7 47.0	+ 9.3	+34.0	3	20 48.9	+43.3	+ 1.0
2	7 50.6	− 6.0	+21.0	5	20 58.5	+12.8	−28.2
4	8 56.6	+10.5	+26.9	5	21 1.3	+31.9	− 2.6
2	8 57.8	+32.0	+25.2	4	21 24.6	−15.8	− 2.9
4	8 58.0	+65.7	− 4.0	3	22 00.2	+12.7	− 7.3
2	9 10.2	+47.2	+19.0	2	22 13.6	+54.7	−14.2
3	9 27.2	− 3.7	+12.3	4	22 32.6	− 6.7	−11.3
2	9 57.2	+22.3	−15.2	5	22 52.9	+25.4	− 0.2
5	10 40.2	−15.2	+19.6	2	23 10.6	+71.4	−26.5
5	10 44.7	+38.1	− 2.8	4	23 10.7	−18.7	+ 5.6
2	11 11.6	+66.1	± 0.0	2	23 16.0	+43.8	− 2.0
6	11 32.8	+ 6.2	+ 3.7	4	23 31.0	+ 5.0	− 0.9

$$\cos D = \sin \delta_0 \sin \delta + \cos \delta_0 \cos \delta \cos (\alpha_0 - \alpha). \qquad (2)$$

If we place

$$\left.\begin{aligned} x &= V \sin \delta_0 \\ y &= V \cos \alpha_0 \cos \delta_0 \\ z &= V \sin \alpha_0 \cos \delta_0 \end{aligned}\right\} \qquad (3)$$

equations (1) take the form

$$\sin \delta . x + \cos \alpha \cos \delta . y + \sin \alpha \cos \delta . z - v = 0 \qquad (4)$$

from which the values of x, y, and z may be determined; and the values of V, α_0, and δ_0 may then be found from (3) by the relations

$$\left.\begin{aligned} V^2 &= x^2 + y^2 + z^2 \\ \tan \alpha_0 &= \frac{z}{y} \\ \sin \delta_0 &= \frac{x}{V} \end{aligned}\right\} \qquad (5)$$

The values of α and δ for each group were substituted in equation (4), and the resulting equation was weighted in proportion to the number of stars on which it is based, as indicated in column 1 of the table. The eighty equations thus formed were combined and solved by the method of least squares, and the following elements of the solar motion were obtained:

$$\begin{aligned} V &= -19.89 \text{ km} \pm 1.52 \text{ km} \\ \alpha_0 &= 277^\circ 30' \pm 4^\circ\!.8 \\ \delta_0 &= +19^\circ 58' \pm 5^\circ\!.9 \end{aligned}$$

The list of stars employed in this investigation includes all that were available: none were rejected arbitrarily, on account of very high speed or otherwise. The results represent the solar motion relative to the entire system of observed stars. Had a dozen stars of great velocity been rejected, the speed and direction of the motion would have been only slightly different, but the computed probable errors would have been very much smaller than those appended.

On the basis of these elements, the component correction for the solar motion was computed and applied to each star. The 280 results obtained in this manner represent the individual stellar components of motion in the line of sight, with reference to the entire system. Of these there are

		Km per second
151	positive, average,	$+17.01$
129	negative, average,	-17.10
280	numerical average,	17.05

The average component velocity of each star in a plane at right angles to the line of sight is therefore

$$\frac{\pi}{2} \cdot 17.05 = 26.78 \text{ km per second};$$

and the average velocity *in space* of each star in the system is

$$2 \times 17.05 = 34.10 \text{ km per second.}$$

The Sun's relative velocity, 19.9 km, is therefore much smaller than that of the average star of the system, 34.1 km.

The 280 stars were classified roughly, according to their spectral types, in the following manner: the *Harvard Photometry* contains estimates of their brightness, based upon the visual radiations. The *Draper Catalogue* estimates their brightness by virtue of the photographic intensities of their spectra in the $H\gamma$ region. The difference between the visual and photographic magnitudes is very small in the case of the white stars, such as β *Orionis;* it is usually from 1.5 to 2.0 magnitudes for the solar type stars, such as β and γ *Andromedae;* and is fully 2.5 magnitudes for red stars, such as α *Scorpii*.

In the system of stars observed, the difference of magnitude is equal to or greater than 1.0 for 144 stars. Subdividing these according as their component velocities in the line of sight are positive or negative, we have

78	positive, average component,	$+ 17.07$ km
66	negative, average component,	$- 14.99$
144	numerical average, - - - -	16.12

For 136 stars the difference of magnitude is less than 1.0, as follows:

73	positive, average component,	$+ 16.94$ km
63	negative, average component,	$- 19.32$
136	numerical average, - - - -	18.04

The discrepancy of 1.9 km in the results is hardly sufficient

to justify any statement as to the effect of spectral type upon velocity.

The relation between visual brightness and velocity was next investigated.

Of stars equal to or brighter than 3.0 magnitude, there are

26 positive, average component,	+ 13.11 km
21 negative, average component	— 12.99
47 numerical average, - - - -	13.05
Corresponding velocity in space, -	26.10

Of stars lying between magnitudes 3.1 and 4.0 inclusive, there are

59 positive, average component,	+ 17.70 km
53 negative, average component,	— 14.42
112 numerical average, - - - -	16.15
Corresponding velocity in space, -	32.30

Of stars fainter than 4.0 magnitude, there are

66 positive, average component,	+ 17.93 km
55 negative, average component,	— 21.27
121 numerical average, - - - -	19.44
Corresponding velocity in space, -	38.88

The progression in these results is so pronounced, and the differences are so large, that I think we are justified in drawing the important conclusion that the faint stars of the system are moving more rapidly than the bright stars. This apparent fact, derived quite independently of any assumption as to the relative distances of the stars of different magnitudes, should profoundly affect the question of, and the methods of determining, the structure of our sidereal system. If the fainter stars are moving relatively more rapidly than has been previously assumed, they must be relatively further from us than the investigations of their proper motions have led us to conclude.

This progression is in no wise due to an increase of probable error of a velocity determination with decreasing magnitude. The probable error of a single determination is well under half a kilometer for such ercellent stars as *Polaris* and *Procyon;* and it is not much greater for fifth magnitude stars whose spectra contain well-defined lines;

The elements of the solar motion deduced above depend upon the assumption that their most probable values are those which make the sum of the squares of the residual stellar components of speed in the line of sight a minimum. This in turn assumes that the magnitudes of these components are distributed according to the law of accidental errors. No doubt they are distributed according to a somewhat different law, which I hope to investigate fully a few years later, before making a definitive determination of the motion, based upon a much larger number of stars distributed over the entire sphere.[1]

The right ascension of the apex, 277°30′, agrees exactly with the value deduced by Professor Newcomb[2] from all the "proper motion" data available; and differs only 1°30′ from Professor Kapteyn's[3] assumed value, 276°. My value of the declination, +19°58′, differs widely from Newcomb's value, +35°, and Kapteyn's, +34°. It must be noticed that very few radial velocities are available for the region −15° to −30°, and none whatever south of −30° declination. Fully one third of the sky is unrepresented in the solution. The data for determining the declination of the apex are extremely unsymmetrical in arrangement. The data north of the line of motion are fairly complete, whereas the data to the south are very incomplete. To determine the declination therefrom is somewhat similar to flying with one wing very imperfect. The right ascension, on the contrary, is determined from data reasonably symmetrical in distribution.

A comparison of my results with those obtained by Stumpe[4] from proper motions is of great interest. He classified the stars of relatively large proper motions according to their visual magnitudes, with the following results for the position of the apex:

[1] An additional reason for delay arises from the fact that many years of observation are required to establish constancy of stellar velocities, in some cases: of the stars used in the above determination, two have since been discovered to have variable velocities.

[2] *Astronomical Journal*, No. 457, pp. 4, 5.

[3] *Astronomische Nachrichten*, No. 3487, p. 104.

[4] *Astronomical Journal*, No. 457, p. 5.

No. of Stars	Magnitude	R. A.	Dec.
284	1 to 5.5	263°.8	+31°.1
473	5.6 to 7.5	290.7	+37.5
238	7.6 to >	286.7	+46.9

In view of these widely different positions of the apex, it is perhaps not surprising that my result for declination, depending upon even brighter stars than his first group, should be smaller than any hitherto obtained.

The motion of the solar system is a purely relative quantity. It refers to specified groups of stars. The results for various groups may differ widely, and all be correct. It would be easy to select a group of stars with reference to which the solar motion would be reversed 180° from the values assigned above. It is perhaps unsafe to draw conclusions, from my value of the declination, concerning the drift of the brighter (and presumably nearer) stars until the data from the southern sky are available.

Before making the preceding solution for the solar motion by the method of least squares, I had already made an approximate determination of the speed of the solar system, by a different method, as follows: the apical distance D of each star was computed from Newcomb's assumed coördinates of that point ($\alpha = 277°.5, \delta = +35°$). The stars were formed into groups according to their apical distances, as indicated in the first column of Table II. The number of stars in each group is given in column two. The mean apical distance of the group is $[D]$ and the mean observed velocity is $[v]$. It is interesting to note that each $[v]$ between apical distances 0° and 90° is negative, and each one between 90° and 180° is positive. Each radial velocity furnishes an equation of condition of the form

$$V - v \sec D = 0, \tag{6}$$

from which to determine V. We shall assume that the weight of each determination is equal to cos D. The resulting value of V will now be given by

$$V = \frac{\Sigma\,(n \cos [D] \,.\, [v] \sec [D])}{\Sigma\, n \cos [D]} = \frac{\Sigma\, n\, [v]}{\Sigma\, n \cos [D]}. \tag{7}$$

Substituting the values of n, $[v]$, and $[D]$ in this equation, I obtained

$$V = -20.4 \text{ km.}$$

TABLE II.

Apical distances	n	$[D]$	$[v]$	$\cos[D]$
0°— 10°	4	7°.4	− 9.9	+0.992
10 — 20	10	15.5	−24.0	+ .964
20 — 30	16	24.7	−17.5	+ .908
30 — 40	24	34.8	−12.9	+ .821
40 — 50	24	44.1	−16.6	+ .718
50 — 60	29	54.6	− 6.1	+ .579
60 — 70	29	64.4	− 7.0	+ .432
70 — 90	47	79.7	− 2.7	+ .179
90 —110	35	99.4	+ 8.0	− .163
110 —120	19	116.3	+14.4	− .443
120 —130	18	124.4	+13.8	− .565
130 —140	10	134.4	+16.4	− .700
140 —150	5	145.1	+14.6	− .820
150 —160	6	156.0	+29.3	− .914
160 —170	4	164.0	+ 6.0	−0.961
	280			

$$V = \frac{\Sigma\, n\, [v]}{\Sigma\, n \cos [D]} = \frac{-3010}{147.5} = -20.4 \text{ km.}$$

If we use this value of V as a basis for further approximations to its true value, by the method of Kapteyn,[1] we shall obtain $V = -19$ kilometers; though it should be said that his method involves assumptions concerning proper motions.

The foregoing data bear decisively upon the question of stellar parallaxes and other fundamental problems; but these portions of the subject are reserved for a future paper.

The work with the Mills spectrograph has furnished many important by-products. Special mention may be made of the discovery of an unexpectedly great number of spectroscopic binaries. Two or more satisfactory observations have been secured for each of 285 stars of my program. From the Mills spectrograph observations alone, we have discovered that thirty-one[2] of these stars are spectroscopic binaries. To these we must add three binaries in the same list previously discovered

[1] *Astronomische Nachrichten*, No. 3487.

[2] Twenty-five of these have been announced in this JOURNAL, and six now await announcement.

by another observer,[1] making thirty-four in all. That is, of 285 observed stars, *more than one star in nine is a spectroscopic binary.* Further, five additional suspected binaries await verification, and it is altogether probable that many other stars in the list are binaries awaiting discovery. Two plates are not sufficient to detect variable velocity, even in many cases of short period; and still less are they sufficient in many cases of long period, now coming to light by virtue of our older observations. It is not improbable that at least one star in five or six will be found to be a spectroscopic binary; and I should not be surprised to see a still larger ratio established.

The proven existence of so large a number of stellar systems differing widely in structure from the solar system gives rise to a suspicion, at least, that our system is not of the prevailing type of stellar systems. The new field of astronomical research thus opened up is of great richness, and may well occupy the attention, for an indefinite period, of the large number of observers and institutions now engaging in its development. It is perhaps unnecessary to say that the measure of success attainable is dependent upon the degree of accuracy[2] realized in the observed velocities.

It is a pleasure to record that I have been assisted most efficiently in these investigations, since August 1897, by Mr. W. H. Wright, assistant astronomer.

LICK OBSERVATORY,
UNIVERSITY OF CALIFORNIA,
December 1900.

[1] DR. BÉLOPOLSKY, at Pulkowa, α_1 *Geminorum;* the well-known variable stars δ *Cephei* and η *Aquilae;* and the independent and prior discovery of the binary character of the well-known variable, ζ *Geminorum.*

[2] In the later observations of the *best stars* with the Mills spectrograph, an extreme range of two kilometers would afford strong suspicion of variable velocity; and the greater portion of a smaller range due to unavoidable errors would arise not from errors in the spectrograms, I believe, but from changes in the observer's personal habits of measuring the plates.

THE MOTION OF ζ *GEMINORUM* IN THE LINE OF SIGHT.

By W. W. CAMPBELL.

ζ *Geminorum* is a well-known variable star, discovered by Schmidt in 1847. It varies from a minimum of 4.5 magnitudes to a maximum of 3.7 magnitudes in 5.015 days, and returns to the minimum in 5.139 days. According to Chandler[1] the period of 10.154 days is well defined, and constant; but Miss Clerke states[2] that the period lengthened by ten minutes between the years 1847 and 1890. The light-curve of ζ *Geminorum* is approximately represented in the upper part of Fig. 1.

This star was placed on the regular observing list for the Mills spectrograph. The second photograph of its spectrum, secured in January 1899, led to the discovery that it is a spectroscopic binary. In response to my announcement of the discovery, published in this JOURNAL for February 1899, Dr. Bélopolsky kindly called attention[3] to his prior discovery of its binary character, in January 1898. He had announced his discovery in a lecture before the Russian Astronomische Gesellschaft; though so far as I am aware no published statement was made.

In the meantime, twenty-two spectrograms had been obtained here, preparatory to determining the orbit of the bright component; and it seemed advisable to continue the observations with the Mills spectrograph.

Forty-four spectrograms were secured between 1898, November 11, and 1900, February 11. The Greenwich mean times of observations are contained in the accompanying table, column one, and the observed velocities, in kilometers, in column three.

[1] "Third Catalogue of Variable Stars," *Astronomical Jour.*, No. 379, pp. 148, 149; and by private letter, June 1900.

[2] *The System of the Stars*, p. 133.

[3] *Astronomische Nachrichten*, No. 3565, May 1899.

	Greenwich Mean Time		Interval after Min.	Observed Velocity	Computed Velocity	O—C
	d	h	d	k	k	k
1900—February	11	16.0	0.017	+21.0 C	+19.0	+2.0
"	11	18.7	0.126	+19.4 C	+18.3	+1.1
1898—November....	11	23.4	0.246	+19.9 C	+17.5	+2.4
1899—February....	21	16.3	0.468	+17.9 W	+16.2	+1.7
"	21	18.2	0.488	+16.3 W	+15.6	+0.7
April........	13	16.8	0.662	+13.3 W	+14.3	−1.0
September ...	13	0.7	0.688	+14.6 C	+14.0	+0.6
October......	23	23.9	1.038	+10.2 W	+11.3	−1.1
"	4	0.9	1.388	+ 5.2 C	+ 8.7	−3.5
February.....	22	16.5	1.417	+ 7.0 W	+ 8.5	−1.5
"	22	18.3	1.492	+ 4.7 C	+ 8.0	−3.3
"	22	18.3		+ 4.7 W	+ 8.0	−3.3
October......	25	0.7	2.071	+ 2.9 C	+ 4.4	−1.5
February.....	13	16.0	2.550	+ 1.7 W	+ 1.9	−0.2
April........	5	15.6	2.767	+ 2.5 W	+ 0.9	+1.6
December....	25	20.6	2.979	+ 1.3 C.	± 0.0	+1.3
January......	24	20.3	3.038	+ 0.7 C	− 0.2	+0.9
December....	26	0.1	3.125	+ 0.7 W	− 0.3	+1.0
1900—January......	15	16.3	3.492	+ 0.5 C	− 1.7	+2.2
1899—December....	26	18.3	3.883	+ 0.0 C	− 2.7	+2.7
"	26	18.3		− 0.2 C	− 2.7	+2.5
January......	25	21.0	4.067	− 1.8 C	− 3.1	+1.3
September ...	27	0.0	4.504	− 3.8 C	− 3.7	−0.1
February.....	15	18.3	4.646	− 4.3 W	− 3.8	−0.5
"	15	19.6	4.700	− 4.7 W	− 3.8	−0.9
December....	27	23.1	5.083	− 6.2 W	− 3.9	−2.3
"	28	0.2	5.129	− 5.9 C	− 3.8	−2.1
1900—February.....	6	20.7	5.367	− 6.7 C	− 3.6	−3.1
1899— "	6	16.6	5.729	− 2.9 W	− 3.0	+0.1
"	6	17.7	5.775	− 2.6 W	− 2.9	+0.3
January......	27	20.2	6.033	− 2.4 C	− 2.1	−0.3
1900—February.....	7	17.2	6.221	− 0.4 C	− 1.4	+1.0
1899— "	7	16.2	6.712	+ 3.6 W	+ 1.2	+2.4
January......	28	20.2	7.033	+ 5.4 C	+ 3.5	+1.9
1900—January......	29	18.4	7.425	+ 8.8 C	+ 7.0	+1.8
1899—November....	30	0.4	7.600	+11.2 C	+ 8.8	+2.4
April........	10	16.4	7.800	+12.6 W	+11.0	+1.6
January......	29	18.4	7.958	+13.0 W	+12.7	+0.3
1900—January......	30	18.1	8.412	+16.2 C	+17.6	−1.4
"	10	19.4	8.775	+18.9 C	+20.6	−1.7
1899—January......	30	19.8	9.017	+20.5 W	+22.0	−1.5
"	30	21.1	9.071	+20.1 W	+22.1	−2.0
1900—January......	21	16.5	9.500	+21.2 C	+22.3	−1.1
1899—September...	22	0.0	9.658	+24.2 W	+21.9	+2.3
"	12	0.5	9.833	+23.5 C	+21.1	+2.4
April........	12	17.8	9.858	+23.5 W	+21.0	+2.5

They are arranged in the order of the intervals after the instants of minima, as indicated in column two. The letters C and W in column three indicate that the plates were measured by Campbell, and Wright, respectively. Some time before the

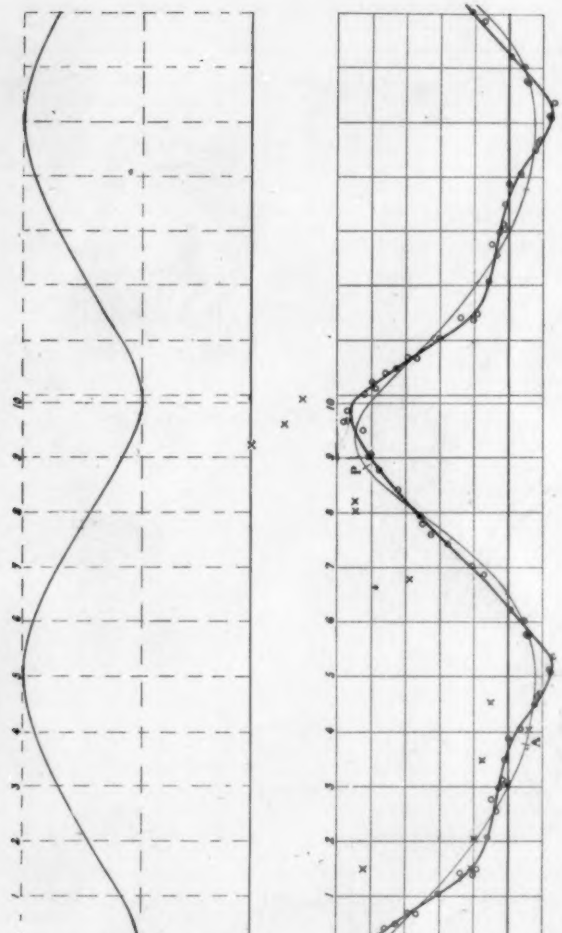

FIG. 1

observations were completed it was noticed that the velocity curve includes some curious irregularities. The later observations were planned, in point of time, so as to fill in the small gaps of the curve; and they leave no doubt that the observed irregularities are real.

In Fig. 1 the unit of abscissæ is one day, and the unit of ordinates is five kilometers. The Mills spectrograph observations are represented by small circles, o; each result being plotted twice in such a way as to show two complete cycles of velocity variations. The heavy irregular line drawn through the observed points is the curve of observed velocities. Its irregular character is well defined, and obviously does not represent motion in an ellipse. After some half a dozen trials of different systems of elliptic elements, the following were adopted, as they appear to afford the best possible elliptic representation of the observed curve.

ELEMENTS.

$V = +6.8$ km, velocity of the system;
$A = 15.7$ km, greatest positive velocity of the bright component;
$B = 10.7$ km, numerical value of greatest negative velocity;
$e = 0.22$, eccentricity of orbit;
$\omega = 333^\circ$, position of periastron;
$T = +1.313$ days, time of periastron passage, referred to the instant of minimum brightness;
$a \sin i = 1{,}797{,}800$ km, projection of semi-major axis.

The velocities computed from these elements for the instants of observation are contained in column four of the table; and the corresponding curve is the lighter line in the lower portion of Fig. 1. There is no satisfactory basis on which to compute the probable error of a single observation; but a simple inspection of Fig. 1 will show that it is in the neighborhood of four or five tenths of a kilometer. Except in good atmospheric conditions, it was difficult to secure a satisfactory spectrogram when the brightness was a minimum.

The observed velocity curve is alternately above and below the elliptic curve, and the intersections of the two occur at

approximately equal intervals of time. There are six of these intersections, corresponding to three complete periods or cycles in exactly one period of the light curve. The observations extend over fifteen months, or about forty-five complete periods; and there is no reason to doubt that the apparent velocity curve repeats itself faithfully during each light period.

It would be possible to explain fairly well the observed irregularities in the velocity by assuming that ζ *Geminorum* is a triple system; that the bright component and a dark companion are revolving around their center of gravity in a period of 3.385 days, with a velocity double amplitude of about 4.5 km for the bright component; and that these two bodies are revolving around a third component in a period of 10.154 days. It is questionable, however, whether such a system would be a stable one. If the short period is exactly commensurate with the longer one, as it appears *on the average* to be, it would seem that the stability of the system would be open to question, especially since the amplitudes 4.5 and 26.4 are not very unequal.

The constancy of the light period, the equality of the light and (apparent) velocity periods, and the satisfactory representation of the *general features* of the velocity curve, leave no doubt, I think, that the system is at least binary, and that the light variations are due in some manner to the influence of the companion. The *form* of the orbit of the bright component, and its relation to the line of sight, are shown in Fig. 2. *O E* is the line of sight, *O* is the center of gravity of the two components of ζ *Geminorum*, *P* is the periastron, and *A* the apastron. The points of the velocity curve corresponding to periastron and apastron are likewise marked *P* and *A*. When the star's brightness is a minimum the bright component is at the point of the orbit marked Min, and the companion is somewhere on the line Om. It is certain, therefore, that the light variation is not the result of an eclipse.

Granting that the variation in brightness is due to the influence of the companion, the most satisfactory explanation available seems to be that it arises from tidal disturbances in the

bright star's atmosphere, produced by the gravitational attraction of the dark component. In all probability the two components are only a few millions of kilometers apart, and their atmospheres may approach comparatively near to each other. Recalling that the eccentricity of the orbit is 0.22, and that the tide-raising force varies inversely as the cube of the distance between the masses, it is plain that the disturbances in the bright star's atmosphere

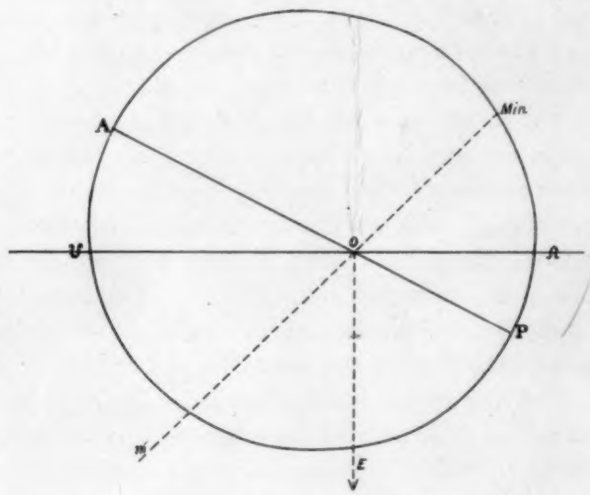

FIG. 2

may easily be on an enormous scale, and be very different in different parts of the orbit.

In this connection attention should be called to the fact that the minimum brightness occurs 1.3 days after periastron passage, and maximum occurs 6.3 days after periastron. It may be urged that if the disturbance is of tidal origin the maximum brightness should occur soon after periastron; and perhaps this is true. But is it not also possible that the maximum tidal forces may lead to a strong expansion of the gases of the atmosphere and to a reduction of brightness? The question must be left in an unsatisfactory state; and a consideration of the phenomena of two other variable stars of the same class, δ *Cephei* and η *Aquilae*, affords

little assistance. In the case of each of these stars the brightness rapidly increases at the time of periastron passage and reaches a maximum very soon thereafter. This is perhaps what would be expected in the case of tidal disturbances, but it is just the opposite of the effect observed in ζ *Geminorum*. However, the orbits of δ *Cephei* and η *Aquilae* are very eccentric, whereas that of ζ *Geminorum* is much less so. The eccentricities in the three cases are 0.46, 0.47, and 0.22, respectively, and their periods are 5, 7, and 10 days. The first two are subjected to vastly more rapid changes of tidal forces than the last; and that fact may account for very divergent effects. It is of interest to note that the brightness of δ *Cephei* and η *Aquilae* varies 1.2 magnitude in five and seven days, respectively, whereas that of ζ *Geminorum* varies only 0.8 magnitude in ten days.

By way of explanation of the deviations of the observed velocity curve from the elliptic curve, I think the probability is very strong that they are minor tidal effects. Terrestrial tides run through their cycle in one period of the Moon — neglecting the second-order quantities in the positions and distances of the Sun and Moon. They are profoundly modified by the rotation of the earth, increasing their number from (roughly) two per month to two per day. If tidal disturbances, with a double period of ten days, are sufficient to account for the light curve of ζ *Geminorum*, I think it is not impossible that the irregularities in its apparent velocity arise from modifications in the tides caused by the rotation of the star. These modifications might affect the apparent velocities in the line of sight in either or both of two ways, viz.:

(A) By producing an actual movement of considerable velocity within the atmosphere; and

(B) By producing considerable variations of pressure within the absorbing layer, and consequent displacement of the spectral lines. The remarkably interesting results obtained by Professor Wilsing, of Potsdam, in his study of the phenomena of "new stars," are sufficient justification for the belief that enormous changes of pressure may occur in such disturbed stars as

ζ Geminorum. In this connection it is very desirable that the light curve of *ζ Geminorum* be determined with great accuracy, to ascertain whether the irregularities in the velocity curve may not have their exact counterparts in the light curve.

These short-period irregularities have not been detected as yet in the velocity curves of *δ Cephei* and *η Aquilae;* but perhaps in these five and seven-day, and very eccentric, systems the periods of revolution and rotation synchronize.

I recognize that the hypotheses advanced above are not now capable of proof, and I have endeavored to state them from that point of view.

Fifteen observations by Dr. Bélopolsky in 1898 and 1899 are plotted in the lower portion of Fig. 1, being represented by an *X.* His observations, on the average, are six kilometers above my curve.

Acknowledgments are due to Mr. Wright for skillful assistance throughout the investigation.

LICK OBSERVATORY,
UNIVERSITY OF CALIFORNIA,
December 1900.

SOME STARS WITH LARGE RADIAL VELOCITIES.

By W. W. CAMPBELL.

WHILE pursuing the regular program of observation with the Mills spectrograph, it was found that the following stars have large velocities in the line of sight, as indicated below:

ϵ ANDROMEDAE ($\alpha = 0^h\ 33^m$, $\delta = +28°\ 46'$).

1898	October 4	− 83.4 km	Wright
	October 9	− 83.3	Wright
1899	August 29	− 84.6	Wright
1900	August 22	− 83.4	Wright
	Mean	− 83.7	

μ CASSIOPEIAE ($\alpha = 1^h\ 0^m$, $\delta = +54°\ 20'$).

1900	September 9	− 97.2 km	Wright
	September 18	− 97.0	Wright
	December 11	− 98	Campbell

The proper motion of μ *Cassiopeiae* is 3″.75 per year. Jacoby's parallax determined from the Rutherfurd photographs is 0″.275. These correspond to a motion at right angles to the line of sight of 66 km per second, though this includes nearly the full component of the motion of the solar system.

δ LEPORIS ($\alpha = 5^h\ 47^m.0$, $\delta = -20°\ 54'$).

1900	December 24	+ 95 km	Campbell
	December 25	+ 96	Campbell
	December 30	+ 94	Campbell

θ CANIS MAJORIS ($\alpha = 6^h\ 50^m$, $\delta = -11°\ 55'$).

1897	December 15	+ 96 km	Campbell
1899	October 16	+ 96.0	Wright
1900	October 9	+ 95.5	Wright

I PEGASI ($\alpha = 21^h\ 17^m$, $\delta = +19°\ 23'$).

1900	July 3	− 75.7 km	Wright
	July 8	− 74.9	Wright
	July 16	− 77.1	Wright

μ SAGITTARII ($\alpha = 18^h\ 7^m.8$, $\delta = -21°\ 05'$).

1899	June 19	— 75 km	Wright
1900	May 30	— 76	Wright

These measures are subject to an uncertainty of several kilometers, on account of the character of the spectrum.

LICK OBSERVATORY,
December 1900.

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The scope of the ASTROPHYSICAL JOURNAL includes all investigations of radiant energy, whether conducted in the observatory or in the laboratory. The subjects to which special attention will be given are photographic and visual observations of the heavenly bodies (other than those pertaining to "astronomy of position"); spectroscopic, photometric, bolometric, and radiometric work of all kinds; descriptions of instruments and apparatus used in such investigations; and theoretical papers bearing on any of these subjects.

In the department of *Minor Contributions and Notes* subjects may be discussed which belong to other closely related fields of investigation.

Articles written in any language will be accepted for publication, but unless a wish to the contrary is expressed by the author, they will be translated into English. Tables of wave-lengths will be printed with the short wave-lengths at the top, and maps of spectra with the red end on the right, unless the author requests that the reverse procedure be followed. If a request is sent *with the manuscript* one hundred reprint copies of each paper, bound in covers, will be furnished free of charge to the author. Additional copies may be obtained at cost price. No reprints can be sent unless a request for them is received before the JOURNAL goes to press.

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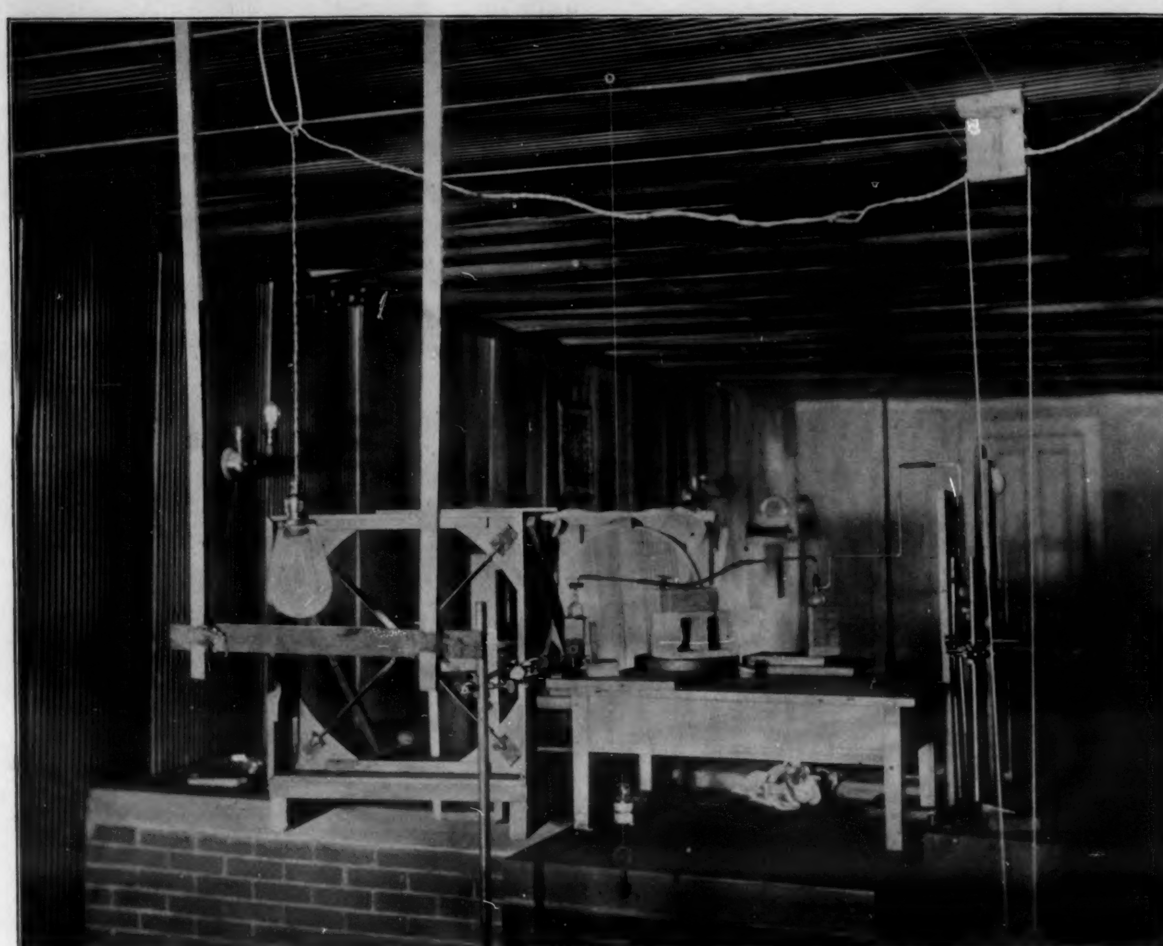
PLATE I.



RADIOMETER AND 24-INCH MIRROR USED IN MEASURING THE HEAT RADIATION OF STARS AND PLANETS.